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ON ACYCLIC CURVES. A SURVEY OF RESULTS AND PROBLEMS

BY J. J. CHARATONIK

Contents

1. Introduction

We start with necessary notation and definitions. All considered spaces are assumed to be metric and all mappings are continuous. We denote by $\mathbb{N}$ the set of all positive integers, by $\mathbb{R}$ the real line, by $\mathbb{I}$ the closed unit interval $[0, 1]$ of reals, and by $\mathbb{S}^1$ the unit circle $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$.

Given a subset A of a space X, we denote its cardinality by $\operatorname{card} A$, its dimension by $\dim A$, its diameter by $\operatorname{diam} A$, its closure by $\operatorname{cl} A$, its interior by $\operatorname{int} A$, its boundary by $\operatorname{bd} A$.

A. Spaces

An *arc* is defined as a homeomorphic image of the interval $\mathbb{I}$, and a *simple closed curve* means a homeomorphic image of the unit circle $\mathbb{S}^1$. A *continuum* means a compact connected space. A *curve* means a one-dimensional continuum. A space is said to be *locally connected* provided that each of its points has an arbitrarily small connected neighborhood. A subset of a space is said to be *arcwise connected* provided that every two of its points can be joined by an arc lying in this set. An arc with end points a and b will be denoted by ab.

A property of a continuum is said to be *hereditary* provided each subcontinuum of the continuum has the property. A continuum X is defined to be

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Key words and phrases: acyclic, arc, confluent, connected, continuum, contractible, curve, dendrite, dendroid, fan, fixed point, locally connected, monotone, open, order, planable, property of Kelley, selectible, smooth, tree-like.

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A REPRESENTATION THEOREM FOR GENERALIZED WIENER PROCESS IN CONUCLEAR SPACE

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1. Introduction

Relations between general martingales and the Wiener process have been investigated by many mathematicians, from several different points of view. One approach consists in embedding a martingale in a Wiener process by means of a random time change (cf.,e.g., [13]), another possibility is to try, for a *fixed* Wiener process, to represent a martingale as the stochastic integral with respect to this process (cf.,e.g., [11, Theorem 5.5]). Still another problem is to *look for* a Wiener process, perhaps in a somewhat augmented probability space, such that a given martingale can be written as the stochastic integral with respect to this Wiener process.

In this paper we adopt the latter approach, which seems particularly important since, besides yielding an insight into the structure of continuous martingales with absolutely continuous Doob-Meyer processes, it constitutes a link between martingale problems and the investigation of weak solutions of stochastic differential equations (see, e.g., [16], [14]). The results for the finite dimensional case are well known (see [16, Theorem 4.5.2]). The Hilbert space case was considered in [17], where a continuous, Hilbert space-valued martingale with an absolutely continuous tensor Doob-Meyer process was represented as the stochastic integral with respect to, suitably constructed, a cylindrical Brownian motion in this space. This result has been applied in [10] to derive a representation theorem for martingales and the Wiener process in the dual of a nuclear Fréchet space. Such a theorem is useful and important, since the duals of nuclear spaces, i.e. conuclear spaces, (typically, the spaces of distributions) are natural state spaces for various limit models, describing some complex physical phenomena, such as, for instance, fluctuations of interacting particle systems (see, e.g., [7], [8], [9]). To investigate these limits an appropriate stochastic analysis apparatus in nuclear spaces must be constructed.

The representation theorem in [10] has been obtained under some additional, rather restrictive assumptions, both on the space and the processes. The principal aim of the present paper is to generalize that theorem. We consider a

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Bol. Soc. Mat. Mexicana (3) vol. 1, 1995

LINEAR PROGRAMMING AND INFINITE HORIZON PROBLEMS OF DETERMINISTIC CONTROL THEORY*

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1. Introduction

The optimal control problem (OCP) we shall be dealing with is the following: Given the infinite-horizon, n-dimensional deterministic control system

(1.1) $$\dot{x}(t) = g(x(t), \mathbf{u}(t)) \qquad \text{if} \qquad t > 0$$

(1.2) $$x(0) = x_0, \qquad x_0 \in \mathbb{R}^n,$$

the objective is to minimize the functional

(1.3) $$J(x_0; \mathbf{u}) = \int_0^\infty e^{-\lambda t} f(x(t), \mathbf{u}(t)) dt$$

over all U-valued control functions $\mathbf{u}$, where $U \subset \mathbb{R}^m$ is a compact set and $\lambda > 0$ is the discount factor. Conditions over the running cost f and the system function g will be given in the following section. The main objective of this paper is to study the linear programming formulation of the OCP (1.1)–(1.3) and present an approximation scheme to OCP through linear programs.

The idea is to embed the OCP in a linear program (P) defined on a space of linear functionals. Using duality theory we introduce the dual program (P^*), which is to find the supremum among all smooth subsolutions of the dynamic programming (or Hamilton-Jacobi-Bellman) equation. We then prove the existence of an optimal solution to (P) and, even though an optimal solution for the OCP is not guaranteed, we show the equivalence of (P) and OCP in the sense that

(1.4) $$\min(P) = V(x_0),$$

where V is the value function of OCP valued at the initial condition x_0 in (1.2). This, in turn, implies that the optimal solution to (P) belongs to the convex closure of the set of feasible solutions to the OCP (in a suitable topology—see Corollary (4.3)).

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Key words and phrases: Discounted cost criterion, linear programming (in infinite dimensional spaces), duality theory, smooth subsolutions, Markov decision processes.

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APPROXIMATION SCHEMES FOR ITÔ-VOLTERRA STOCHASTIC EQUATIONS*

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1. Introduction

As shown in [2] stochastic equations of Itô-Volterra type can be used as models for systems perturbed by noise.

Existence or existence and uniqueness theorems for such equations can be found in [1], [3], [4], [8], [9], [14], [16] in the diffusion case and in [7], [13], [15], [17] for semimartingale differentials.

However, as in the case of Itô equations, explicitly solvable Itô-Volterra equations are rare in practical applications. For this reason it becomes important to find approximation schemes which can be used for the simulation of the paths of the solution. We mention the work by Makroglou [10], where collocation methods as applied to deterministic Volterra integro-differential equations are extended to solve stochastic Volterra integro-differential equations.

In the present paper we consider a general strong approximation scheme for Itô-Volterra equations in the diffusion case (Theorem (2.1)). This extends to Volterra equations a result obtained in [11] for Itô equations. The general formulation in Theorem (3.1) is complemented by some examples which are introduced in section 4.

2. Some Preliminaries

Let $\{W_t\}_{t\geq 0} = \{(W_t^1, \ldots, W_t^m)\}_{t\geq 0}$ be a m-dimensional Brownian motion and let $\{\varphi(t)\}_{t\geq t_0}$, $t_0 \geq 0$, be a càdlàg and $\mathscr{F}_t$-adapted real valued process defined on a complete filtered probability space $(\Omega, \mathscr{F}, P, \{\mathscr{F}_t\}_{t\geq 0})$.

Let $c(t, s, x)$: $R_+^2 \times R^d \to R$ be a continuous function such that:

(i) $c(t, t, x)$ is Lipschitz in x, uniformly in t.

(ii) $c_1(t, s, x) := \frac{\partial c}{\partial t}(t, s, x)$ is Lipschitz in x, uniformly in (t, s).

For $0 \leq j \leq m$ define the process $\{\tilde{L}_c^j(s, \varphi)\}_{s\geq t_0}$ by

$$\tilde{L}_c^j(s, \varphi) = \int_{t_0}^{s} c(s, s_1, \varphi(s_1)) dW_{s_1}^j \tag{2.1}$$

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