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OBSTRUCTIONS TO PROPAGATION OF GROUP ACTIONS

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The most basic questions in the theory of transformation groups involve understanding which manifolds admit group actions and to what extent one can vary the basic invariants of an action on a given manifold. These problems are nowhere near a complete resolution.

When we restrict our attention to finite groups, we encounter a fundamental phenomenon in the subject that goes back to the early work of P.A. Smith. It is possible to obtain homotopical information concerning the action quite well at the order of the group. Away from the order of the group, the relationships tend to be much coarser. With propagation and replacement, one tries to systematically study the homological phenomenon just described. That is, we try to understand when manifolds with the same $\mathbb{Z}/|\pi|$-homological type have the same theory of π-actions, and we try to analyze the extent to which manifolds which are of the same $\mathbb{Z}/|\pi|$-homological type can be interchanged as fixed sets.

In this paper, we will concentrate on propagation results, although we hope to apply these results to replacement elsewhere. For technical reasons involving homotopy theory, most propagation problems deal with *homologically trivial* group actions, that is, actions which induce the identity on homology with coefficients in $\mathbb{Z}[1/|\pi|]$. We will also restrict our attention to free actions, because they are critical to any induction. [C-W] and [D-L] give a complete analysis for odd order group actions. In [D-W1] we implicitly gave an analysis (or at least a reduction to explicit algebraic number theoretical problems) in the case of free actions of arbitrary finite groups assuming that the map between manifolds has degree one and is in a homotopy theoretical sense tangential. See [A-V], [J], [L-R], [Q2] for other works on propagation and [We] for a survey.

In [D-W2], we apply the techniques in this paper to semifree actions on the sphere, and in particular obtain new restrictions on fixed point sets in the smooth case. These ideas were also applied in [We2] to complete the classification of semifree *PL* locally linear fixed point sets on the sphere for orientation preserving actions (assuming no codimension two fixed point sets). Indeed these papers justify our expectation that our methods have something to say, quite generally, about replacement.

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ON THE K-THEORY OF BIANCHI GROUPS

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Abstract

In this paper, we compute the ring structure of the p-adic K-theory of the classifying space of the so-called "Euclidean Bianchi groups", we use a multiplicative formula found by one of the authors that applies to this situation and the geometry associated to these groups.

Introduction

Let $\mathcal{O}_d$ denote the ring of integers in a quadratic number field $k = \mathbb{Q}(\sqrt{d})$ with $d \in \mathbb{Z}$, $d < 0$, and square free. Let Γ_d denote the group $PSL_2(\mathcal{O}_d) = SL_2(\mathcal{O}_d)/\pm I$. These are known as the Bianchi groups. In this work we compute the ring structure of the p-local K-theory of the classifying space $B\Gamma_d$ of Γ_d, where $d = -1, -2, -3, -7, -11$ and p is a prime number.

We apply a formula found by one of the authors in [J2]. This applies when Γ is a group of finite virtual cohomological dimension which satisfies suitable finiteness conditions. Let $\Gamma' \subseteq \Gamma$ be a normal torsion free subgroup of Γ of finite index. In this situation the formula reads

$$\text{(I)} \qquad \mathbb{Q}_p \otimes K_p^*(B\Gamma) \cong \bigoplus_{(C)} [\mathbb{Q}_p[\xi_c] \otimes K_p^*(B(N_\Gamma(C) \cap \Gamma'))]^{W_C},$$

where $\mathbb{Q}_p$ denotes the p-adic rationals, K_p the K theory with coefficients in the p-adic integers [A], C a finite cyclic p-subgroup Γ of order c, (C) a set of representatives for the Γ-conjugacy classes of finite cyclic p-subgroups, ξ_c a c-th primitive root of unity, $\mathbb{Q}_p[\xi_c]$ its corresponding cyclotomic extension, and $W_C = N_\Gamma(C)/N_\Gamma(C) \cap \Gamma'$, we will refer to this as the W-group of C.

A key step in our computations is to understand the abelian subgroups of these Bianchi groups. This is partially done in [Fi], and we complete Fine's classification:

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LOCAL PROPERTY OF VISCOSITY SOLUTIONS OF FULLY NONLINEAR SECOND ORDER ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

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Abstract

The two approaches to viscosity solutions of fully nonlinear elliptic pde's, one as introduced in Caffarelli and Wang and the other as in Crandall, Ishii and Lions, are studied. For some class of uniformly elliptic operators, the first notion is shown to be local and is equivalent to the second one.

1. Introduction

In this note, we study the two approaches to the viscosity solution of fully nonlinear second order elliptic pde's: one approach, similar to that developed in Caffarelli and Wang ([1], [5]) and the other one, as in Crandall, Ishii and Lions [2]. We show that the first notion is local and is equivalent to the second one for some class of uniformly elliptic operators, thus establishing a link between the two approaches.

2. Viscosity solutions

Let Ω be a nonempty open bounded subset of $\mathbb{R}^n$ and S_n be the subspace of all symmetric $n \times n$ matrices. Let F be a continuous map from $S_n \times \mathbb{R}^n \times \mathbb{R} \times \Omega$ to $\mathbb{R}$, satisfying the uniform ellipticity condition:

$$\Lambda\|M\| \geq F\,(N+M, p, z, x) - F\,(N, p, z, x) \geq \lambda\|M\|, \tag{2.1}$$

for all $N \in S_n$, $M \geq 0$ and $(p, z, x) \in \mathbb{R}^n \times \mathbb{R} \times \Omega$. Here $\|M\| = (\sum_j \sum_i |m_{ij}|^2)^{1/2}$ for the matrix $M = [m_{ij}]$. First we study the viscosity solutions of

$$F(D^2u(x), Du(x), u(x), x) = 0 \tag{2.2}$$

in the spirit of [1] and [5].

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ON THE DYNAMICS OF THE ONE PARAMETER FUNCTIONS $F_a(z) = z^2 + 2a\bar{z}$

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Abstract

We associate the set $K(F_a)$, to the family of functions $F_a(z) = z^2+2a\bar{z}$, where $z \in \mathbb{C}$ and $a \in \mathbb{R}$, $K(F_a)$ is the set points in $\mathbb{C}$ whose orbit under F_a is bounded. We describe the bifurcations of F_a and some of its dynamics on $K(F_a)$, focusing mainly on the connectedness of $K(F_a)$.

Introduction

The quadratic mappings $f_c(z) = z^2 + c$, $z \in \mathbb{C}$ have been studied by many authors (Douady, Hubbard, Yoccozz, et. al), the dynamics of this family of holomorphic maps is encoded by the well-known Mandelbrot set. In fact, if $J(f_c)$ denotes the Julia set for the above maps, the set of parameter values c for which $J(f_c)$ is connected defines the Mandelbrot set.

More recently J. Milnor, R.Winters [7] and others have studied the equivalent to the Mandelbrot set for the family of antiholomorphic maps defined by $g_c(z) = \bar{z}^2 + c$.

On the other hand G. Gómez and S. López de Medrano studied from the dynamical point of view a classification of families of quadratic maps (with singularities) from $\mathbb{R}^2$ to $\mathbb{R}^2$, see [3]. In their work, they ask to what extent the behaviour of the dynamics of holomorphic mappings can be extended to non-holomorphic maps. One of the families in the classification given in [3] is $F_a(z) = z^2 + 2a\bar{z}$, $a \in \mathbb{R}$, for which the authors constructed computational images of $J(F_a)$ (see Definition 2) for some values of a.

The above family $F_a(z)$ shares with the holomorphic family $f_c(z)$ the fact that the singular set of both functions is compact and that ∞ is an attractive fixed point (see Lemma (3)).The singular set of $F_a(z)$, is a circle (see §0), while the singular set of $f_c(z)$ is a point. Moreover, $F_a(z)$ is a universal unfolding ($a \in \mathbb{C}$) for the map $F_0(z) = z^2$ (see the Appendix)

In this paper we investigate the connectivity of $J(F_a)$, proving that $J(F_a)$ is connected if and only if $a \in [-1, 2]$ (see theorems (1), (2), (3)). As in the complex case, the singular set of F_a plays an important role in proving the

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STRONG SOLUTIONS OF ANTICIPATING STOCHASTIC DIFFERENTIAL EQUATIONS ON THE POISSON SPACE

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Abstract

In this paper we use the Poisson-Itô chaos decomposition approach to study existence and uniqueness of strong solutions of anticipating stochastic differential equations driven by a compensated Poisson process. As a tool, we introduce another type of solution which allows us to construct the strong solution using the Picard-Lindelöf iteration procedure.

1. Introduction

The extension of the Itô stochastic integral on the Wiener space to anticipating integrands has allowed to study stochastic differential equations where the initial condition and the coefficients may be anticipating (see Pardoux [12] and references therein). For the extension of the integral, there are basically two different approaches. The first method uses the Wiener chaos decomposition (see Section 2) and the second one is based on the definition of a "derivative" operator D and on the derivation of an integration by parts formula (see Nualart and Pardoux [8]).

On the Poisson space, the above methods give different definitions of integral (see, for instance, [1], [2], [5], [9], [10]).

In this paper we use the Poisson-Itô chaos decomposition approach to study existence and uniqueness of strong solutions of anticipating stochastic differential equations driven by a compensated Poisson process $\widetilde{N}$ on the canonical Poisson space. Namely, the form of these equations is formally written as

$$dX_t = b(t, X_t)\,dt + B(t, X_t)\,\delta\widetilde{N}_t, \quad t \in [0, T], \tag{1.1}$$

where b, $B\colon \Omega \times [0, T] \times R \to R$.

As a tool, we introduce another type of solution of equation (1.1) (see Definition (3.3)), which allows us to construct the strong solution by induction on the number of jumps of the paths of the compensated Poisson process $\widetilde{N}$.

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