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REVIEW OF SOME RECENT RESULTS IN PERTURBATION THEORY FOR SOLITARY WAVES

ANTONMARIA A. MINZONI

Abstract

The purpose of this review is to present some recent results in the perturbation theory of solitary waves. Results for integrable and non integrable systems are visited. The examples are from different fields of application. Some of the examples concerning the interaction of solitons and radiation are discussed in more detail. Also ideas which could be of wider applicability are developed. Unsolved problems are presented and the status of rigorous results relevant to perturbation theory is also discussed.

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AN EXTENSION ON STETCHKIN'S CONDITION

JOSÉ M. GONZÁLEZ BARRIOS[1] AND BELEM TREJO VALDIVIA

Abstract

In the late 1920's Copson proved an inequality relating the behavior of a series of positive terms to that of the series generated by the tails, later on Stetchkin proved an equivalence for the convergence of two series for the case "$p = 1/2$", while working with absolute convergence of Fourier series. In this note we give an inequality in the other direction of Copson's inequality in the case of decreasing sequences, which extends Stetchkin's result.

1. Introduction

Copson extended some theorems of Hardy and Littlewood concerning series of positive terms. In his papers [1], [2] he proved that given a sequence $\{c_n\}$ of positive terms, and $0 < p < 1$ the following inequality holds.

$$\sum_{n=1}^{\infty}\Big(\frac{c_n + c_{n+1} + \cdots}{n}\Big)^p > p^p \sum_{n=1}^{\infty} c_n^p.$$

The aim of this note is to give an inequality in the other direction to extend Stetchkin's result for $0 < p < 1$. Stetchkin gave a condition for a Fourier series to be absolutely convergent using Copson's inequality for the case $p = 1/2$, see [7],[9] and [10], we will prove that a similar condition holds for $0 < p < 1$.

1. Main results

Given a sequence $\{a_n\}_{n=1}^{\infty}$ of positive terms which decreases to 0 we will use the following Theorem, whose proof follows the lines of Theorem 1.5 in [3] in order to obtain the other inequality.

THEOREM (1.1). *If $\{a_n\}_{n\geq 1}$ is a sequence of real numbers such that $a_n \downarrow 0$, that is, a_n decreases to 0, and $b_n := (\sum_{j\geq n} a_j^{1/p})^p$ where $0 < p < 1$, then the following are equivalent:*

a) $\sum_j a_j < \infty$.

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Keywords and phrases: Stechkin's condition, Copson's inequality, trace class operators.
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INTERCALATE MATRICES: I. RECOGNITION OF DYADIC TYPE

GILBERTO CALVILLO, ISIDORO GITLER, AND JOSÉ MARTÍNEZ-BERNAL

Abstract

This is the first of a series of papers where we study the characterization problem of which intercalate matrices determine integral formulas of sum of squares. Matrices that can be embedded into the Cayley table of a group of exponent two are intercalate matrices, called dyadic. This paper is devoted to giving combinatorial criteria to recognize dyadic matrices.

1. Introduction

The aim of the present paper is to characterize which intercalate matrices can be embedded into the Cayley table of a group of exponent two, such matrices are called dyadic. To this end we give five criteria: the first two hinge on a simple theorem of alternatives given in Section 2. Lemmas (3.1) and (4.1) found below rely on the fact that an intercalate matrix is completely determined by the position of its intercalations (or co-intercalations); their proofs are not given, the reader should have no problem in proving them, as well as proving Lemmas 5.1 and 5.2. The other criteria are based on connectedness, homogeneity and duality of intercalate matrices, respectively. All of these concepts are defined further below.

In this paper we consider the group of exponent two $(\mathbb{N}, \oplus)$, where $\mathbb{N} = \{0, 1, 2, \dots\}$ and $\oplus$ is the **dyadic sum** defined as follows: consider the binary representation of a and b, then add componentwise mod2 to obtain $a \oplus b$. We regard $\mathbb{N}$ as a vector space over $GF(2)$.

A matrix is **intercalate** if its entries, thenceforward called colors, along any row and along any column are all distinct, and furthermore, each of its 2×2 submatrices has an even number of distinct colors. Those 2×2 submatrices with two (four) distinct colors are called **intercalations** (**co-intercalations**).

The Cayley table of $(\mathbb{N}, \oplus)$ is an infinite intercalate matrix that we denote by $D[\mathbb{N} : \mathbb{N}]$. For (ordered) subsets $R = \{r_1, \dots, r_p\}$ and $C = \{c_1, \dots, c_q\}$ of $\mathbb{N}$, denote by $D[R : C]$ the $p \times q$ submatrix of $D[\mathbb{N} : \mathbb{N}]$ whose (i, j)-color is $r_i \oplus c_j$; R and C are called the set of row and column generators of $D[R : C]$, respectively.

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Keywords and phrases: group of exponent two, intercalation, intercalate matrix, dyadic matrix, isotopy, ill aligned, sum of squares formulas.

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REDUCTION THEOREMS FOR UNIFORM STABILITY OF SYSTEMS IN GENERAL SPACES

PETER SEIBERT, J. HÉCTOR ARREDONDO,
JOAQUÍN DELGADO AND LUIS AGUIRRE

Abstract

A theory of reduction of stability properties of a not necessarily compact invariant set to a subsystem defined on a subspace of lower dimension is presented in the context of a dynamical or semidynamical system on a metric space.

Introduction

The idea of reducing a problem of stability of a system of differential equations defined on a state space X to a similar problem formulated for a system defined on a space Y of lower dimension has been treated already in Lyapunov's famous mémoire [8], and in one case even earlier by Poincaré [12]. Afterwards the problem was studied extensively by the Russian school of stability[1], an outstanding contribution being that of V.A. Pliss [13], who first introduced the idea of what today is known under the name of *centre manifold*.

A major limitation of these earlier contributions consists in the fact that, depending strongly on the linear part of the system, they are strictly local. Also, in the case of systems with highly degenerate linear part (for instance in the case of absence of linear terms), they are not applicable at all. These considerations have given rise to a more general problem of reduction which may be phrased in the following terms: Suppose a system defined on a state space X possesses an invariant subspace Y (of lower dimension than X) which itself contains a closed invariant subset M, and that M is known to have a certain stability property (such as Lyapunov stability, asymptotic stability, global asymptotic stability) with respect to the reduction of the original system to Y. Then the question which arises is the following: What dynamical conditions

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Keywords and phrases: Lyapunov stability, reduction principle for stability of not necessarily compact invariant sets in abstract spaces.

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[1]For an account of these historic antecedents, see the introduction of the research report [5]

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A QUASILINEAR DIFFERENTIAL GAME OF EVASION

MANUEL J. FALCONI

Abstract

A sufficient condition for evasion in a quasilinear differential game is obtained. The proof is based on the structure of the solutions. Although our condition reflects a certain superiority of evader, it is simpler than the sufficient conditions assumed by analogous works recently published.

1. Introduction

In this work the following differential game is considered. Suppose we have the equation

$$\dot{z} = F(z, u, v), \tag{1.1}$$

with $u \in P, v \in Q,\ z \in \mathbb{R}^n$ and P and Q are subsets of $\mathbb{R}^n$. For each $u(t) \in P$, we set as problem to obtain a value $v(t) \in Q$ in such a way that the solution of

$$\dot{z}(t) = F(z(t), u(t), v(t)); \quad z(0) = z_0, \tag{1.2}$$

does not intersect M (M is a target subspace of $\mathbb{R}^n$), for any $t \geq 0$. Some evader's games can be stated in this form, see for example [2] and [3].

In [3] the linear case was studied. There

$$F(z, u, v) = Cz - u + v, \tag{1.3}$$

with C is a $n \times n$-square matrix. The given conditions for C, P and Q, guarantee that $z(t)$ never touches M. In this paper a generalization of this work is provided. We do not assume a linear dependence for the controls u and v; instead, we take the system

$$\dot{z} = Cz + F(u, v), \tag{1.4}$$

where F is a continuous function. Our result includes the linear case of [3] and is complementary to recent works in this area, (see [4]-[8]). The Theorem

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Keywords and phrases: Differential Game, Game of Evasion, Evadability.

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AN INVERSE PROBLEM FOR NONLINEAR SOURCE TERMS

JOHN R. CANNON AND PAUL C. DUCHATEAU

Abstract

In this article we consider the problem of identifying the unknown source term $F = F(u)$ in the operators $u_t - u_{xx} = F(u)$ and $-\Delta u = F(u)$ from overspecified initial/boundary data. We show for functions u admitting formal power series expansions $u = \sum_{i=0}^{\infty} \varepsilon^i u_i$ and $F(u) = \sum_{i=0}^{\infty} \varepsilon^i F_i(u)$, the problem reduces to a sequence of simpler inverse problems whose solutions may be constructed by straightforward methods. The convergence and stability of the perturbation series is not considered.

0. Introduction

We consider here the problem of identifying the unknown source term $F = F(u)$ in the parabolic partial differential equation

$$u_t - u_{xx} = F(u)$$

or its stationary counterpart, the elliptic equation

$$-\Delta u = F(u),$$

where Δ is the Laplace operator in x and y. The problem of identifying an unknown source term of this form has significant applications to problems in chemical kinetics and combustion, to problems in population dynamics and in modelling neutron diffusion in nuclear reactors.

Here F is to be identified from overspecified data measured on the boundary of the region where the equation applies. The natural measurements at the boundary include the Dirichlet data,

$$u = f \text{ on } \partial\Omega,$$

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Keywords and phrases: Inverse Problems, Parabolic Equations AMS (MOS).

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ON THE EULER-MACLAURIN FORMULA

RICARDO ESTRADA

Abstract

Using the distributional theory of asymptotic expansions, new Euler-Maclaurin type formulas for numerical quadratures based on generalized functions are derived. The expansions for functions with fractional power behavior at the endpoints is obtained. The development for functions with discontinuities is also considered and the appearance of oscillatory functions as coefficients is observed.

1. Introduction

It is common knowledge that integrals can be approximated by sums and conversely. One of the earliest formal results on such approximations is the celebrated Euler-Maclaurin formula.

In its basic form [5], the Euler-Maclaurin formula states that if a function ϕ is of class C^q in an interval $[N, M]$, where N, M are integers, then

$$\sum_{k=N}^{M} \phi(k) = \int_N^M \phi(x)dx + \frac{1}{2}(\phi(N) + \phi(M)) + \sum_{j=2}^{q} \frac{(-1)^j B_j}{j!}(\phi^{(j-1)}(M) - \phi^{(j-1)}(N)) + R_q, \tag{1.1}$$

where the remainder $R_q = R_q(\phi)$ is given by

$$R_q = \frac{(-1)^{q-1}}{q!} \int_N^M B_q(x - [\![x]\!])\phi^{(q)}(x)dx. \tag{1.2}$$

Here the B_j are the Bernoulli numbers and $B_j(x)$ are the Bernoulli polynomials, defined in Section 3. Equation (1.1) is a very precise stament of the vague approximation $\sum_{k=N}^{M} \phi(k) \sim \int_N^M \phi(x)dx$.

The Euler-Maclaurin formula has found applications in several contexts. It has been generalized in many ways and many variants have been given. An

1991 *Mathematics Subject Classification*. 41A60, 46F10.
Keywords and phrases: Asymptotic expansion, numerical quadrature, generalized function.

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MAXIMAL IDEAL SPACE FOR CLASSICAL DOMAINS

GIOVANNA CARCANO

Abstract

We determine the maximal ideal space for commutative algebras of integrable functions on the Šilov boundary of classical generalized half-planes, which are invariant under the action of particular "rotation groups"; this is realized by suitably decomposing the representation spaces.

Introduction

Let $\mathscr{D}$ be a Siegel domain of type II and $\mathscr{N}$ be the nilpotent Lie group identified to its Šilov boundary. It is interesting to consider the Banach algebra $\mathscr{A}$ of integrable functions on $\mathscr{N}$, which are invariant by some automorphisms of $\mathscr{N}$ (they are the natural generalization of the *radial* functions). By [3], we know in which classical cases $\mathscr{A}$ is commutative; in this paper, we determine the *maximal ideal space* for such commutative algebras (i.e. the set of all non-zero multiplicative linear functionals on $\mathscr{A}$). To realize this, we have to *concretely* decompose the representation spaces $\mathscr{H}_\pi$ (π representation of $\mathscr{N}$) into irreducible subspaces with respect to the action of an *intertwining* representation; this implies the consideration of the *whole* dual space (and not only the *reduced* dual).

Our paper is organized as follows: in §1, we describe the general situation (domains $\mathscr{D}$, algebras $\mathscr{A}$, how the multiplicative functionals are obtained, etc.) and we prove that the maximal ideal space depends only on the space of orbits in $\hat{\mathscr{N}}$ (Lemma (1.8)); in §2 we consider $\mathscr{D} = SU(p, p+q)/S(U(p)\times U(p+q))$: the case p=1 ($\mathscr{N}$=Heisenberg group) was studied in [7] and [8], the general case is carried out decomposing the representation Fock space by means of spaces of polynomials annihilated by a particular differential operator (this is comparable to the decomposition of $L^2(\mathbb{R}^n)$ in terms of solid spherical harmonics ([12], Chap.IV §2); in §3 we consider $\mathscr{D} = SO^*(2n)/U(n)$.

The tools mainly used in the paper are the representation theory of the Šilov boundary and of the classical groups, the "character" and the "spectral" formulas.

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Keywords and phrases: Siegel domains of type II, Šilov boundary, Gelfand pair, Gelfand transform, representation Fock space.

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THE (Γ, t)-TOPOLOGY ON $L(E, E)$ AND THE SPECTRUM OF A BOUNDED LINEAR OPERATOR ON A LOCALLY CONVEX TOPOLOGICAL VECTOR SPACE

RIGOBERTO VERA MENDOZA

Abstract

In this paper we introduce a different topology than the most common ones defined on the vector space $L(E, E)$ of all linear and continuous operators with domain and codomain the topological vector space $E[t]$. We prove some results using this topology. Then, we define, for each $T \in L(E, E)$, the real number $\gamma(T)$ in terms of the new topology and we prove some facts about this number and its resemblance with the spectral radius of T.

1. Introduction

E will denote a vector space over the field **C** of the complex numbers and t will denote a Hausdorff, locally convex, topology on E such that $E[t]$ is a toplogical vector space. We shall also assume that $E[t]$ is complete.

$L(E, E)$ will be the linear space over **C** of all linear and continuous operators $T: E[t] \to E[t]$.

We start our discussion by recalling the basic facts about nets and by showing how a net of operators in $L(E, E)$ converges over a family of nets in E. Then, we discuss the topology on $L(E, E)$.

Notation. Let $\vartheta(t)$ be the family of all balanced, convex, closed neighborhoods of zero in E.

We shall work with nets in our topological vector spaces.

A net $\{x_\alpha\}_J \subset E$ is a map from a directed set $(J, \leq)$ into E. We refer the reader to [KotI] for a discussion of this concept and the associated terminology.

We shall need the following concepts:

(a) A net $\{x_\alpha\}_J \subset E$ is said to be a Cauchy net if given any $V \in \vartheta(t)$ there is an $\alpha_0 \in J$ such that $x_\alpha - x_\beta \in V \ \forall \alpha, \beta \geq \alpha_0$.

(b) A net $\{x_\alpha\}_J \subset E$ is convergent to an element $y \in E$ if given any $V \in \vartheta(t)$ there is an $\alpha_0 \in J$ such that $y - x_\alpha \in V \ \forall \alpha \geq \alpha_0$.

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Keywords and phrases: ultimately bounded (ub-net), (Γ, t)-closed, (Γ, t)-convergent, (Γ, t)-Cauchy, (Γ, t)-bounded..

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PSEUDOCHARACTERS AND UNIFORM WEIGHTS

ADALBERTO GARCÍA-MÁYNEZ

Abstract

If X is a Tychonoff space, we define two weights $\omega' X$, $\omega'' X$ which depend on the cardinalities of the uniformity basis on X which are compatible with the topology of X. These new weights are related with the classical weight ωX of X (i.e., the minimum cardinality of a basis for X). We introduce also the notion of strong pseudocharacter $\psi'(A, X)$ of a subset $A \subset X$ and exhibit some connections between ωX, $\omega' X$, $\omega'' X$, $\psi'(\Delta(X), X \times \beta X)$, the pseudocharacter of the Dieudonné completion μX in the Stone-Čech compactification βX and the cardinality of the family of zero sets in $X \times X$.

Let A be a subset of a topological space X. A set $U \subset X$ is called a *strong neighborhood* of A if there exist a zero set H and a cozero set D such that $A \subset H \subset D \subset U$. The *strong pseudo-character* $\psi'(A, X)$ is defined as the minimum cardinal α such that A is the intersection of α strong neighborhoods. The *strong weight* $\omega' X$ of a completely regular space X is defined as the minimum cardinality of a compatible uniformity basis of X. A uniformity basis $\mathscr{U}$ in X is *coherent* if every z-ultrafilter in X which is Cauchy in $(X, \mathscr{U})$ is also Cauchy in $(X, \mathscr{U}_n)$, where $\mathscr{U}_n$ is the uniformity basis of X consisting of all normal covers of $(X, \tau_{\mathscr{U}})$. Observe $\mathscr{U}_n$ itself is coherent and every complete uniformity basis in X is coherent. The *coherent weight* $\omega'' X$ of a completely regular space X is defined as the minimum cardinality of a compatible coherent uniformity basis of X.

If X is a Tychonoff space, we prove the equality $\omega' X = \psi'(\Delta(X), X \times \beta X)$ and exhibit some connections between ωX, $\omega' X$, $\omega'' X$, the pseudocharacter of the Dieudonné completion μX in the Stone-Čech compactification βX and the cardinality of the family of zero sets in $X \times X$.

1. Definitions and preliminary results

Unless otherwise stated, no separation axiom will be assumed. We start with some definitions.

1991 *Mathematics Subject Classification*. 54A25, 54E15.

Keywords and phrases: strong neighborhood, strong pseudo-character, strong weight, coherent uniformity basis, coherent weight, scale, connector, pseudometric cover, equivalent cozero covers..

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APPROXIMATION THEOREMS WITH A-CONVERGENCE

José Luis Fernández Muñiz

Abstract

In this paper we study sequences of linear operators which are "almost positive". For them, we prove Bohman-Korovkin type theorems for convergence in A-norm and the test families of functions which the author introduced in a previous paper.

1. Introduction

Let X be a compact Hausdorff topological space. By $F(X)$ (respectively $B(X)$, $C(X)$) we denote the linear space of complex (or real) functions on X (respectively bounded, continuous). By E we denote a linear subspace of $F(X)$ or $B(X)$.

In [2] we introduced the following definitions.

Definition (1.1). Let (L_n), $n \in N$, be a sequence of linear operators from E into $B(X)$. We say that (L_n), $n \in N$, is a sequence of class $\widetilde{R}$, if for every $\epsilon > 0$, there exists $N(\epsilon) \in N$ such that for every $n \geq N(\epsilon)$, for every f in E with $\mathrm{Re} f \geq 0$, and for every $t \in X$ we have $\mathrm{Re} L_n(f, t) > -\epsilon$.

Definition (1.2). We say that (f_x), $x \in X$, is a *test family* of functions in E if the following conditions hold:

(a) for every x in X, $f_x \in E$, and the function $(x, t) \mapsto f_x(t)$ is continuous in $X \times X$;
(b) for every x in X, $\mathrm{Re}\, f_x(x) = 0$;
(c) for every t in X, $t \neq x$, $\mathrm{Re}\, f_x(t) > 0$.

For these kinds of sequences and test families, and equivalently for contracting linear operators, we presented in [2] some Korovkin type results for the uniform convergence, supported by two fundamental lemmas. Later, in [3] we proved analogous fundamental lemmas for the convergence in A-norm,

1991 *Mathematics Subject Classification*. 41A10, 41A25, 41A28.

Keywords and phrases: compact Hausdorff topological space, sequence of $\tilde{R}$ class, test family of functions, Korovkin type results, A-norm, A-modulus of continuity, A-continuous function, sublinear functional, A-equicontinuous sequence, continuously A-convergent sequence.

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