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NATURAL DUALITIES FOR SOME SUBVARIETIES OF DMS

R. SANTOS

Abstract

A natural duality is obtained for any variety of double *MS*-algebras. These dualities allow us to describe free algebras.

1. Preliminaries

Algebras $(L; \wedge, \vee, \circ, 0, 1)$ of type $(2, 2, 1, 0, 0)$ such that $(L; \wedge, \vee, 0, 1)$ is a bounded distributive lattice and $\circ$ is a dual endomorphism of $(L; \wedge, \vee, 0, 1)$ are called Ockham algebras and form a variety.

A double **MS**-algebra $(L;\wedge,\vee,\circ,+,0,1)$ is an algebra of type $(2, 2, 1, 1, 0, 0)$ such that $(L; \wedge, \vee, \circ, 0, 1)$ and $(L; \wedge, \vee, +, 0, 1)$ are Ockham algebras and $\circ, +$ satisfy

$$x \leq x^{\circ\circ},\ x^{++} \leq x,\ x^{\circ+} = x^{\circ\circ},\ x^{+\circ} = x^{++},\ \forall x \in L.$$

These algebras were introduced by T. Blyth and J. Varlet in [1], inspired by the properties of double Stone algebras. **DMS** will denote the variety of double **MS**-algebras.

The 22 non-isomorphic subdirectly irreducible algebras of **DMS** were described in [2]. It will be convenient to have their diagrams present in mind to completely understand the meaning of the symbols we use in what follows.

It is known from [3, pg. 1] that if $\underline{A}$ and $\underline{B}$ are subdirectly irreducible algebras in **DMS**, then $\underline{A} \leq \underline{B}$ iff $\underline{A} \in \mathbf{HS}(\underline{B})$ iff $\underline{A} \in \mathbf{S}(\underline{B})$. As a consequence, using Jónsson's Lemma and Birkhoff's Subdirect Product Theorem, we can say that for each subdirectly irreducible algebra $\underline{A}$ in **DMS** the variety generated by $\underline{A}$, $\mathbf{HSP}(\underline{A})$, is equal to $\mathbf{ISP}(\underline{A})$.

We must recall that M. Sequeira in [14] presents a Priestley duality for a class of algebras which includes **DMS**-algebras. This duality proved very useful to completely describe the free algebras of the varieties we are studying.

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Keywords and phrases: natural duality, double *MS*-algebra.

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INTERCALATE MATRICES: II. A CHARACTERIZATION OF HURWITZ-RADON FORMULAS AND AN INFINITE FAMILY OF FORBIDDEN MATRICES

GILBERTO CALVILLO, ISIDORO GITLER, JOSÉ MARTÍNEZ-BERNAL

Dedicated to the memory of our dear friend François Jaeger

Abstract

For a long time it has been known that there exists a sums of squares formula of type (r, n, n) with integer coefficients if and only if $r \leq \rho(n)$, where ρ is the Hurwitz–Radon function. In this paper we characterize, by means of intercalate matrices, those formulas of type $(\rho(2^n), 2^n, 2^n)$ that use only integer coefficients. We also construct an infinite family of nonsignable intercalate matrices which are (inclusionwise) minimal with this property.

1. Introduction

Intercalate matrices are combinatorial objects related to sums of squares formulas of type

$$(r, s, n) \equiv (x_1^2 + \cdots + x_r^2)(y_1^2 + \cdots + y_s^2) = z_1^2 + \cdots + z_n^2,$$

where each z_k is an integral bilinear form in the indeterminates x and y; that is, $z_k = \sum a_{i,j}^k x_i y_j$ with the $a_{i,j}^k$ integers. In fact, there is a canonical correspondence between signable intercalate matrices of type (r, s, n) and sums of squares formula of type (r, s, n) that use only integer coefficients (see [23], [27], [29]).

It is well known (see for example [2], [7], [9], [14], [18]) that:

(1.1) There exists a formula of type (r, n, n) with integer coefficients if and only if $r \leq \rho(n)$, where ρ is the Hurwitz-Radon function given by

$$\rho(n) = 8a + 2^b; \quad n = 16^a 2^b c, \quad 0 \leq a,\ 0 \leq b \leq 3,\ c \text{ odd}. \tag{1.2}$$

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Keywords and phrases: binary matroid, bipartite matroid, dyadic matrix, forbidden matrix, free subset, group of exponent two, Hurwitz–Radon function, intercalate matrix, odd circuits, sums of squares formula.

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SOME REMARKS ON THE JACOBIAN VARIETY OF PICARD CURVES

ALEXIS GARCÍA ZAMORA

Abstract

Given a Picard curve X, a set of equations defining an affine open set of $\operatorname{Jac} X$ is obtained. We prove that a Jacobian variety of dimension 3 satisfying certain theta-vanishing properties in 1/6-periods is necessarily the Jacobian of a Picard curve.

1. Introduction

In this paper by a variety we shall mean a separable, irreducible and reduced scheme of finite type over $\mathbb{C}$. A point is always a closed point. We use freely the fact that an algebraic variety over $\mathbb{C}$ has a natural structure of a complex analytic space.

Definition (1.1). Let X be a non-singular, non-hyperelliptic irreducible curve of genus 3. X is said to be a *Picard curve* if there exists a cyclic group $\langle\sigma\rangle \subset Aut(X)$ of order 3 such that $X/\langle\sigma\rangle \simeq \mathbb{P}^1$.

Remarks. (1.1.1). Given a non-singular, non-hyperelliptic curve X of genus three we think of X as its image under the canonical embedding. Thus, we identify X with a plane, non-singular curve of degree 4.

(1.1.2). It is easy to see that the cover $\pi: X \to \mathbb{P}^1$ induced by $\langle\sigma\rangle$ is Galois, so all the ramification points of π are of degree of ramification 2. By Riemann-Hurwitz it follows that there are 5 of these points. We call these $R_1, \ldots, R_5$.

(1.1.3). In a suitable choice of affine coordinates the equation of X can be written as

$$y^3 = \prod_{i=1}^{4}(x - a_i).$$

Here $R_i = (a_i, 0)$, $i = 1, .., 4$ and R_5 is identified with the point at infinity, denoted by ∞.

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THE REPETITIVE ALGEBRA OF A GENTLE ALGEBRA

CLAUS MICHAEL RINGEL

Abstract

We are going to show that the repetitive algebra of a gentle algebra whose quiver has at least two cycles is non-domestic (thus even of non-polynomial growth).

1. Introduction

Let k be a field. Given a quiver Q and arrows $\alpha: a \to b, \beta: b \to c$, the concatenation of α and β will be denoted by $\alpha\beta$, it is a path of length 2 starting in a and ending in c. Recall that a relation for the quiver Q is a non-zero linear combination of paths of length at least 2 having the same starting point and the same end point.

Let Q be a connected quiver and ρ a set of relations for Q. The pair (Q, ρ) is said to be *special biserial* provided the following conditions are satisfied:

(1) Every vertex is starting point of at most two arrows.
(1′) Every vertex is end point of at most two arrows.
(2) For every arrow β, there is at most one arrow α with $\alpha\beta \notin \rho$.
(2′) For every arrow β, there is at most one arrow γ with $\beta\gamma \notin \rho$.

The pair (Q, ρ) is said to be *gentle* provided besides (1), (1′), (2), (2′) also the following conditions are satisfied:

(3) All the relations in ρ are monomials of length two.
(4) For every arrow β, there is at most one arrow α' with $\alpha'\beta \in \rho$.
(4′) For every arrow β, there is at most one arrow γ' with $\beta\gamma' \in \rho$.

A k-algebra A is said to be *special biserial*, or *gentle*, provided A is isomorphic to the factor algebra $kQ/\langle\rho\rangle$, where (Q, ρ) is special biserial, or gentle respectively; here kQ denotes the path algebra of the quiver Q, and $\langle\rho\rangle$ the ideal generated by the set ρ. (The k-algebras which we consider will always have sufficiently many idempotents, but not necessarily a unit element; note that the path algebra of a quiver Q has a unit element if and only if Q has

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Keywords and phrases: representations of finite dimensional algebras, quivers, special biserial algebras, gentle algebras, repetitive algebras, representation type: tame, domestic, polynomial growth. strings and bands.

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DIRAC OPERATORS ON SPHERES AND HYPERBOLAE

JOHN RYAN

Abstract

Cayley type transforms are used to show that a significant portion of Clifford analysis developed over R^n can be transfered to the sphere and to the hyperbola. In particular we show that the conformal weight functions needed to describe the conformal covariance of solutions to the Euclidean Dirac equation are linearized with respect to the Pin group acting on the hyperbola. This leads to a simplification of aspects of the existing theory. Extensions of complex Clifford analysis to the complex sphere can also be described in this context. In particular we give a description of the Lie ball within the complex sphere.

0. Introduction

The use of Vahlen matrices to describe the links between Dirac operators over R^n, or C^n, and the conformal group has been pursued and applied by a number of authors, see for instance [C, PQ, QR, R3]. In this paper we use Vahlen matrices to describe two types of Cayley transformations. One is from R^n to the punctured sphere lying in R^{n+1}, while the other is from the open unit disc in R^n to the hyperbola in the Minkowski/Krain space $R^{n,1}$. Using these Cayley transformations it is possible to show that much of existing Clifford analysis over Euclidean space can carry through to these settings.

Dirac operators over general manifolds have been utilised in a number of contexts, see for instance [GiMu, LaMi, B-BW]. In [C] an attempt is made to draw Clifford analysis closer to this more general setting. In this paper we show that using the special tools of conformal geometry one can achieve quite a bit within the class of examples of manifolds considered here. In particular one can set up a Cauchy integral formula, Plemelj projection operators, and so decompose certain L_p spaces over a wide class of surfaces into Hardy spaces over the respective complementary domains of the surface. This essentially enables one to show that results set up in the Euclidean setting, over Lipschitz surfaces in [LMcQ, LMcS, M] and elsewhere, carry over to the context described here. This in part fits in with some results on Calderon projections described in [B-BW] for the surfaces described here.

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Keywords and phrases: Dirac operators, Clifford algebra, Vahlen matrices, Cayley transformations.

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STABILITY OF DIFFERENTIAL EQUATIONS VIA THE THEORY OF CONTINUED FRACTION EXPANSIONS

ZIAD ZAHREDDINE

Abstract

New continued fraction expansions related to stable systems of differential equations are obtained. They are expressed in terms of an arbitrary function and in some special cases they reduce to continued fraction expansions of classical types. Properties of some of the terms involved in these expansions are also studied, and an illustrative example is given. The established continued fraction expansions may be used to test the stability of a system of differential equations in either the discrete—or the continuous—time case.

1. Introduction

Stability analysis of systems of differential equations is fundamental to the analysis of many mathematical problems. For example, the stability or instability of a given system will be determined by the roots of its characteristic polynomial. Based on the Sturm theorem and the Cauchy index, Routh derived an algorithm and set it in a tabular form-called now the Routh Array-for determining the number of eigenvalues of a real system in the open left-half plane. Also based upon the determinants of a sequence of matrices, Hurwitz gave an alternative method to achieve the same objective as Routh did. The well-known Routh-Hurwitz criterion becomes an extremely important tool in studying the stability of linear continuous-time systems, and their works have been applied in many fields. As some major references in this respect we cite [3], [4], [5], [9] and [14]. In [13], a new and different approach to the stability problem from an analytic perspective was advanced, and in [7, Ch.16, Sec.14], special cases of the present work may be found.

Throughout this work, a stable continuous-time system of differential equations is a system that has all its eigenvalues in the left-half of the complex plane, and a stable discrete-time system is one that has all its eigenvalues inside the unit disc. An interesting connection between continued fraction expansions and systems that are stable with respect to the left-half plane has been established in [12, theorem 3.2]. This theorem played a key role in the

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Keywords and phrases: continued fraction expansions, stability analysis, continuous-time systems, discrete-time systems.

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SOLVING SINGULAR NONLINEAR TWO POINT BOUNDARY VALUE PROBLEMS

Rafael G. Campos

In memory of my friend Rogelio Muñoz

Abstract

A very simple Galerkin-collocation-type method for solving singular nonlinear two point boundary value problems with homogeneous boundary conditions is presented in this work. This method is based on the use of a matrix representation of the differential operator d/dx connected to Lagrangian interpolation and defined, element by element, in terms of the zeros of the Jacobi polynomial $P_N^{(\alpha,\beta)}(x)$, which are found to be a convenient choice for the nodes.

In this paper we give a new asymptotic estimate for the remainder between the derivative of a function $f \in C^1(-1,1)$ belonging to the Sobolev class $W^1[-1,1]$ evaluated at the nodes and the elements of the vector obtained by multiplying the Lagrangian matrix representing d/dx and the vector formed with the values of the function at the nodes. This error estimate is related to the N-th coefficient of the Fourier expansion of f in terms of the Jacobi polynomials.

This approximation scheme is applied to elaborate an algorithm for solving singular boundary value problems in one variable, and an expression for the error yielded by this technique is found. This result shows the high-order accuracy of the method.

1. Introduction

The basis of a certain numerical method for solving two point boundary problems was given in a series of papers [1,3–9]. This Galerkin-collocation-type technique yields highly accurate approximants to some differential eigenvalue problems of the second order in one dimension [1] and it can be used to solve without substantial additional effort, a boundary value problem of order $2M$. Such a method consists basically in the substitution of the differential operator

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Keywords and phrases: numerical differentiation, error analysis, singular nonlinear boundary value problems.

NEUTRICES AND THE NON-COMMUTATIVE NEUTRIX PRODUCT OF DISTRIBUTIONS

BRIAN FISHER AND ADEM KILIÇMAN

Abstract

The non-commutative neutrix product of the distributions $x_+^r \ln x_+$ and x_+^{-s} is evaluated for $r, s = 1, 2, \dots$. Further non-commutative neutrix products are deduced.

In the following, we let N be the neutrix, see van der Corput [1], having domain $N' = \{1, 2, \dots, n, \dots\}$ and range the real numbers, with negligible functions finite linear sums of the functions

$$n^\lambda \ln^{r-1} n, \quad \ln^r n: \qquad \lambda > 0, \quad r = 1, 2, \dots$$

and all functions which converge to zero in the normal sense as n tends to infinity. A function $f(n)$ is then said to have the neutrix limit L as n tends to infinity if $f(n) - L$ is a negligble function and we then write $\operatorname*{N-lim}_{n\to\infty} f(n) = L$.

Neutrix limits can be used to define functions and distributions. For example, it was proved in [5] that the Gamma function $\Gamma(x)$ can be defined for $x < 0$ by

$$\Gamma(x) = \operatorname*{N-lim}_{n\to\infty} \int_{1/n}^{\infty} t^{x-1} e^{-t}\, dt, \quad x \neq -1, -2, \dots$$

and the distribution x_+^λ can be defined for $\lambda < -1$ by

$$\langle x+^\lambda, \phi(x)\rangle = \operatorname*{N-lim}_{n\to\infty} \int_{1/n}^{\infty} x^\lambda \phi(x)\, dx, \quad \lambda \neq -2, -3, \dots$$

for arbitrary test function ϕ. However, in the following, we use neutrix limits to define products of distributions.

We now let $\rho(x)$ be any infinitely differentiable function having the following properties:

(i) $\rho(x) = 0$ for $|x| \geq 1$,
(ii) $\rho(x) \geq 0$,

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A NOTE ON PAIRS OF PROJECTIONS

N.J. KALTON

Abstract

We give a brief proof of a recent result of Avron, Seiler and Simon.

In [1], it is proved that if P, Q are (not necessarily self-adjoint) projections on a Hilbert space and $(P-Q)^n$ is trace-class (i.e. nuclear) for some odd integer n then $\operatorname{tr}(P-Q)^n$ is an integer and in fact, if P and Q are self-adjoint, $\operatorname{tr}(P-Q)^n = \dim E_{10} - \dim E_{01}$ where $E_{ab} = \{x : Px = ax,\ Qx = bx\}$; (see also [2]). The proof given in [1] uses the structure of the spectrum of $P-Q$ and Lidskii's theorem; it is therefore not applicable to more general Banach spaces. The purpose of this note is to give a very brief proof of the same result which involves only simple algebraic identities and is valid in any Banach space with a well-defined trace (i.e. with the approximation property). We use $[A, B]$ to denote the commutator $AB - BA$.

The basic material about operators on Banach spaces which we use can be found in the book of Pietsch [3]. We summarize the two most important ingredients.

We will need the following basic result from Fredholm theory. Suppose X is a Banach space and $A: X \to X$ is an operator such that for some m, A^m is compact. Let $S = I - A$; then $F = \bigcup_{k\geq 1} S^{-k}(0)$ is finite-dimensional and if $Y = \bigcap_{k\geq 1} S^k(X)$ then Y is closed and X can be decomposed as a direct sum $X = F \oplus Y$. Furthermore F and Y are invariant for S and S is invertible on Y. We refer to [3] 3.2.9 (p. 141-142) for a slightly more general result.

We will also need the following properties of nuclear operators and the trace. If X is a Banach space then an operator $T: X \to X$ is called nuclear if it can written as a series $T = \sum_{n=1}^{\infty} A_n$ where each A_n has rank one and $\sum_{n=1}^{\infty} \|A_n\| < \infty$. The nuclear operators form an ideal in the space of bounded operators. When X has the approximation property, one can then define the trace of T *unambiguously* by $\operatorname{tr} T = \sum_{n=1}^{\infty} \operatorname{tr} A_n$ (where the trace of a rank one operator $A = x^* \otimes x$ is defined in the usual way by $\operatorname{tr} A = x^*(x)$.) The trace is then a linear functional on the ideal of nuclear operators and has the property that $\operatorname{tr}[A, T] = 0$ if A is bounded and T is nuclear. See Chapter 4 of [3] and particularly Theorem 4.7.2.

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MONOTONE-OPEN MAPPINGS OF RATIONAL CONTINUA

JANUSZ J. CHARATONIK AND WŁODZIMIERZ J. CHARATONIK

Abstract

It is proved that each monotone-open nonconstant mapping defined on a rational continuum is a homeomorphism. As a consequence it follows that each rational continuum which is homogeneous with respect to monotone-open mappings is a simple closed curve.

1. Introduction

Very recently a number of new results concerning continuous monotone decompositions of continua appeared in the literature (see e.g. [8], [9], [10], [11], [12]). Existence of such decompositions, which is equivalent to the existence of a monotone and open mapping defined on the considered continuum, is related to homogeneity with respect to the class of monotone-open mappings. Recall that, given a class $\mathfrak{M}$ of mappings between continua (which contains homeomorphisms and is closed with respect to compositions), a topological space is said to be *homogeneous with respect to* $\mathfrak{M}$ provided that for very two points in the space there is a mapping in $\mathfrak{M}$ of the space onto itself that maps one of the points to the other. In [2,Theorem 5, p. 131] the first named author has shown that the Sierpiński universal plane curve is homogeneous with respect to monotone mappings, and asked if it is homogeneous with respect to open ones ([2, Problem 1, p. 130 and Problem 3, p. 132]). An affirmative answer to this question was given even in so strong form that the curve is homogeneous with respect to mappings which are monotone and open simultaneously, [8, Corollary 25] and [12, Theorem 15]. Since open mappings between locally compact spaces do not increase Menger-Urysohn order of a point (see e.g. [14, Corollary (7.31), p. 147]), homogeneity of a continuum with respect to open mappings forces the continuum to have the same Menger-Urysohn order of a point at all points of the continuum, see [3, Proposition 2, p. 492]. It is an old result of P. S. Urysohn saying that if all points of a continuum are of the same order $\mathfrak{n}$, then this order can take only four values, namely $\mathfrak{n} \in \{2, \omega, \aleph_0, \mathfrak{c}\}$, see [13, Chapter VI, Section 2, p. 105] and compare [3, Proposition 4, p. 493]. If locally connected plane curves are under consideration, then the examples of such continua with the same order of all points are known. Namely, for

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Keywords and phrases: continuum, homogeneous, mapping, monotone, open, rational.

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m-CONFLUENT MAPS, AN EXAMPLE

ALEJANDRO ILLANES

Abstract

For each $n \geq 3$, we construct hereditarily decomposable continua X and Y and a map $f: X \to Y$ such that Y does not contain n-ods and f is not $(2n-5)$-confluent, i.e., there exists a subcontinuum M of Y such that M is not the image of $2n-5$ components of $f^{-1}(M)$. This answers a question by E.E. Grace and E.J. Vought.

Maćkowiak has proved [2, (6.12), page 53] that if Y is an atriodic continuum and f is a map from a continuum X onto Y, then f is 2-confluent. Nall and Vought have shown [5, page 5] that, if Y is a graph, $n \geq 2$ is an integer such that Y contains an n-od but no $(n+1)$-od, and f is a map from a continuum X onto Y then f is $\left[\frac{3}{2}n-1\right]$-confluent and there are examples that show that $\left[\frac{3}{2}n-1\right]$ cannot be reduced for any n. Nall [4, Theorem II.2, p. 412] considered the more general situation where $n \geq 3$ is an integer and Y is any continuum that does not contain and n-od, and stated that f is then $(n-1)(n-2)$-confluent. Grace and Vought [1] improved this result by showing that f is $(2n-4)$-confluent and they gave an example for each n where the number $2n-4$ cannot be lowered. Their example was constructed using a pseudoarc and they asked [1, Question 4] if it is possible to construct such an example with hereditarily decomposable continua. In this paper we answer Grace and Vought's question by constructing, for each $n \geq 3$, hereditarily decomposable continua X and Y and a map $f: X \to Y$ such that Y does not contain n-ods and f is not $(2n-5)$-confluent.

A *continuum* is a compact connected metric space. A continuum Y is an *n-od*, where n is an integer greater than 1, if Y contains a subcontinuum A such that $Y - A$ has at least n components. A *map* is a continuous function. If f is a map from a continuum X onto Y and n is a positive integer, then f is *n-confluent* (= n partially confluent [4, page 409]) if each continuum M in Y is the union of the images of n or fewer components of $f^{-1}(M)$. Note that 1-confluent maps are also called weakly confluent ones.

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Keywords and phrases: m-confluent maps, continuum, n-*od*.

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OPEN BOOK DECOMPOSITIONS ASSOCIATED TO HOLOMORPHIC VECTOR FIELDS

JOSÉ SEADE

Abstract

In this article we follow the ideas of Milnor in [12] to construct open book decompositions associated to holomorphic vector fields. The construction is similar to the one for complex singularities done by Milnor.

Introduction

In 1968 Milnor [12] proved that if $f: (\mathscr{U} \subset \mathbb{C}^n, \mathbf{0}) \to (\mathbb{C}, \mathbf{0})$ is the germ of a holomorphic function with a single critical point at $\mathbf{0}$ and $\mathscr{V} = f^{-1}(\mathbf{0})$, then for every sufficiently small sphere $\mathbb{S}_\varepsilon$ around $\mathbf{0}$, the map

$$\phi: \mathbb{S}_\varepsilon - \mathscr{V} \to \mathbb{S}^1,$$
$$z \to \frac{f(z)}{\|f(z)\|},$$

is the projection map of a C^∞-fibre bundle. In the last section of his book Milnor studied real mappings from $\mathbb{R}^{n+k}$ into $\mathbb{R}^k$. He showed that if f is a submersion on a punctured neighbourhood of $\mathbf{0} \in \mathbb{R}^{n+k}$, then f gives rise (essentially) to a fibre bundle,

$$\phi: \mathbb{S}_\varepsilon^{n+k-1} - \mathscr{V} \to \mathbb{S}^{k-1}$$

as before, though the projection map ϕ may not be the obvious one $\frac{f(z)}{\|f(z)\|}$, c.f. [7]. In page 100 of his book Milnor points out that the weakness of this fibration theorem for real singularities is the difficulty of finding examples satisfying the hypothesis. In particular he asks whether there exist *non-trivial* examples with $k = 2$; this was answered positively by Looijenga in [9], by proving that for every $n > 1$ there exists a real polynomial map $(\mathbb{R}^{2n}, \mathbf{0}) \to (\mathbb{R}^2, \mathbf{0})$ defining a fibration of Milnor's type, which is not topologically equivalent to the real map underlying any holomorphic function from $(\mathbb{C}^n, \mathbf{0})$ into $(\mathbb{C}, \mathbf{0})$. (See also [1,8,14]). However, these results do not throw much light into the problem of studying real singularities for which one has a fibration of Milnor's type [12].

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LIMIT THEOREMS FOR TRANSFORMATIONS OF SUMS OF IID RANDOM VARIABLES

JOSÉ A. VILLASEÑOR ALVA

Abstract

This note contains some results on the limiting distribution of $\varphi\left(\sum_{i=1}^{n} X_i\right)$ suitably normalized where $X_1, X_2, \ldots$ is a sequence of iid random variables with finite variance and φ is a well behaved function at infinity. The imposed conditions on φ are satisfied by most common functions.

1. Introduction

Let $X_1, X_2, \ldots,$ be a sequence of independent identically distributed (iid) random variables with mean μ and variance $\sigma^2 < \infty$. Define the random variables S_n by the sum of the first n variables X_i's, that is

$$S_n = \sum_{i=1}^{n} X_i\,.$$

The Central Limit Theorem states that the limiting distribution of S_n properly normalized as n tends to infinity is normal (0, 1). That is, there exist sequences of constants $a_n > 0$ and b_n such that

$$\lim_{n\to\infty} P\left((S_n - b_n)/a_n \le x\right) = \Phi(x) \tag{1.1}$$

where $\Phi(x)$ is the normal (0, 1) distribution function and x is any real number.

In some instances it is of interest to know the limiting distribution of a transformation $\varphi(S_n)$ suitably normalized as n tends to infinity.

This problem has been studied in [5] and then in [2] for the particular case when the common cumulative distribution function (cdf) of the X_i's is exponential. Their results are related to the asymptotic distribution of the n-th record value of a sequence of iid random variables.

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Keywords and phrases: limiting distribution, regular variation, central limit theorem, transformations, sums.

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GENERALIZED VARIATIONAL QUASI INTERPOLANTS IN $(H_0^m(\Omega))^n$

M. N. BENBOURHIM AND P. GONZÁLEZ-CASANOVA

Abstract

In this paper we aim to construct a scattered vectorial quasi-interpolant, capable of approximating fluid flow data. We base our construction on a generalized energy that includes a rotational, a divergence and a strain term. The approximating scheme is defined on the space $(H_0^m(\Omega))^n$, where $H^m(\Omega)$ is the usual Sobolev space and Ω is an open bounded subset of $\mathbb{R}^n; n = 2, 3$. We also prove convergence to the data function as the data points becomes dense in Ω. Moreover, since its characterization is explicit, the quasi-interpolant is suitable for use in practical applications.

1. Introduction

When considering the approximation of vectorial fields, a key problem is how to correlate its components. It has been observed, particularly for meteorological problems, that if no inter-component correlation is assumed the approximating surface may give unrealistic results, see [7]. In [1] a minimizing vectorial interpolation problem is stated and solved. The minimizing energy used in this last article has a divergence and a rotational term, each multiplied by a fixed real positive constant that controls its relative weight. The size of the constants depends on the particular field to be considered. In [2], by using the stored energy of an isotropic hyper-elastic material, an interpolating formula was constructed in order to approximate vectorial fields with an inter-component correlation controlled by such energy. One of the most important limitations of the interpolating schemes of radial type, see [5] for scalar cases, is that its computation frequently requires to solve a non sparse and bad conditioned linear system of equations. Some preconditioning algorithms have been formulated to overcome this problem, see [9] for instance. However since the range of this system is equal to the number of data, its computation can be extremely time consuming particularly for three dimensional problems.

Quasi-interpolation methods based on radial basis functions, present a different approach to the approximation problem. Unlike for the interpolation

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