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Contenido

ARTÍCULO PANORÁMICO

Harmonic and Riemannian foliations
J. Eells and A. Verjovsky 1

ARTÍCULOS DE INVESTIGACIÓN

The zero divisors of the Cayley-Dickson algebras over the real numbers
G. Moreno 13

Expansion in Newton interpolation series and U-Laplace transform
D. Duverney 29

Convolution multipliers on weighted Besov spaces
J. Ansorena and O. Blasco 47

Quasiconformal distortion of univalent functions via holomorphic motions
L. Reséndis 69

Upcrossing inequalities for powers of nonlinear operators and Chacon processes
S. E. Ferrando 85

Continúa/Continued on back cover

SOCIEDAD MATEMATICA MEXICANA

Bol. Soc. Mat. Mexicana (3) Vol. 4, 1998

HARMONIC AND RIEMANNIAN FOLIATIONS

JAMES EELLS AND ALBERTO VERJOVSKY

Abstract

We survey the geometric aspects of Riemannian and harmonic foliations of smooth manifolds.

1. Introduction

A foliation $\mathcal{F}$ of a manifold M is a decomposition into submanifolds (called *leaves*), governed by a completely integrable system of partial differential equations (Theorem of Frobenius). If M is a complete Riemannian manifold, then its metric structure determines special foliations with rich geometry. Such *Riemannian foliations* are characterized as having locally equidistant leaves. If the leaves of $\mathcal{F}$ are closed, then the quotient space $M/\mathcal{F}$ is a Riemannian orbifold; and the quotient map $\pi : M \to M/\mathcal{F}$ is harmonic iff the leaves of $\mathcal{F}$ are minimal submanifolds of M.

Our aim is to provide a gentle survey of these Riemannian geometric aspects, with examples-without proofs, but with guides to the literature. For background references, we recommend Molino [M 2], and Tondeur [T].

All manifolds and structures on them are assumed smooth ($=C^\infty$) throughout.

2. Foliations

(2.1) A *foliation of* M is a partition of M, $\mathcal{F} = \cup\{F_y : y \in N\}$ into a union of disjoint connected p-dimensional submanifolds—indexed by a set N—for which there is an atlas $\{(\theta, U)\}$ of M such that the induced partition of U into connected components, F_y^U are of the form

$$\theta : F_y^U \to c \times \mathbb{R}^p,$$

for all $y \in N$ and some $c \in \mathbb{R}^q$. Here $\mathbb{R}^m = \mathbb{R}^q \times \mathbb{R}^p$. We call F_y *a leaf of* $\mathcal{F}$. We shall always assume $0 < p, q < m$.

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Keywords and phrases: Fibrations, foliations, Riemannian foliations, harmonic foliations and morphisms.

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THE ZERO DIVISORS OF THE CAYLEY-DICKSON ALGEBRAS OVER THE REAL NUMBERS

GUILLERMO MORENO

Abstract

In this paper we describe the zero divisors of the Cayley-Dickson algebras $\mathbb{A}_n = \mathbb{R}^{2^n}$ for $n \geq 4$.

1. Introduction

The Cayley-Dickson algebra $\mathbb{A}_n$ over $\mathbb{R}$ is an algebra structure on $\mathbb{R}^{2^n}$ given inductively by the formulae:

Let $x = (x_1, x_2)$ and $y = (y_1, y_2)$ be in $\mathbb{R}^{2^n} = \mathbb{R}^{2^{n-1}} \times \mathbb{R}^{2^{n-1}}$. Then, following the standard notation (see [1] or [8])

$$xy = (x_1 y_1 - \overline{y}_2 x_2, y_2 x_1 + x_2 \overline{y}_1)$$

where $\overline{x} = x$ for $x \in \mathbb{R}$ and $\overline{x} = (\overline{x}_1, -x_2)$.

Therefore $\mathbb{A}_0 = \mathbb{R}$, $\mathbb{A}_1 = \mathbb{C}$ (complex numbers), $\mathbb{A}_2 = \mathbb{H}$ (the Hamiltonian quaternions), $\mathbb{A}_3 = \mathbb{O}$ (the Cayley octonions), etc. These four algebras: $\mathbb{R}, \mathbb{C}, \mathbb{H}$ and $\mathbb{O}$ are known as the classical Cayley-Dickson algebras and their distinctive feature is:

HURWITZ THEOREM. *Let* $\| \;\; \|$ *denote the euclidean norm in* $\mathbb{R}^{2^n}$. *Then* $\|xy\| = \|x\|\|y\|$ $\forall x$ *and* y *in* $\mathbb{A}_n$ *if and only if* $n = 0, 1, 2, 3$. (See [5] and [7]).

That is, for $n \geq 4$, this norm-preserving formula is not true in general and this opens the possibility of the existence of *zero divisors* in $\mathbb{A}_n$ for $n \geq 4$; i.e., non-zero elements x and y in $\mathbb{A}_n$ such that $xy = 0$. For example, let $x = e_1 + e_{10}$ and $y = e_{15} - e_4$ in $\mathbb{R}^{16} = \mathbb{A}_4$, where $e_0, e_1, e_2, \ldots, e_{15}$ is the canonical basis in $\mathbb{R}^{16}$.

In this paper we study the zero divisors in $\mathbb{A}_n$ for $n \geq 4$. On the algebraic side, these algebras are non-commutative for $n \geq 2$ and non-associative for $n \geq 3$. Moreover $\mathbb{A}_3$ is *alternative*: $x^2 y = x(xy)$ and $xy^2 = (xy)y$ for all x and y in $\mathbb{A}_3$, and for $n \geq 4$, $\mathbb{A}_n$ is *flexible*:

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EXPANSION IN NEWTON INTERPOLATION SERIES AND U-LAPLACE TRANSFORM

DANIEL DUVERNEY

Abstract

We give a general criterion for developing an entire function f in a Newton interpolation series $\sum_{n=0}^{+\infty} A_n(x-v_1)(x-v_2)\dots(x-v_n)$, where (v_n) is a given sequence of complex numbers. This criterion uses the notion of U-exponential type, and its proof involves the U-Laplace transform, whose properties are studied.

1. Introduction

Let (v_n), $n = 1, 2, 3, \dots$, be a sequence of complex numbers, and put

$$P_0(x) = 1 \ ; \ P_n(x) = (x-v_1)(x-v_2)\dots(x-v_n) \quad \text{for } n \geq 1. \tag{1.1}$$

The purpose of this paper is to give a simple condition allowing us to develop an entire function f in a *Newton interpolation series*

$$f(x) = \sum_{n=0}^{+\infty} A_n \, P_n(x). \tag{1.2}$$

This problem has been studied since Polya's work [24] on integer-valued entire functions, by Gel'fond [16], Pisot [23], Bundschuh ([9], [10]), Walliser [28],..., in the special cases of $v_n = n$ or $v_n = q^{n^\alpha}$ $(|q| > 1)$, by using the exponential type, the q-exponential type, or the logarithmic order of the entire function f. For recent results on integer-valued entire functions, see Bézivin's work [6].

In order to study this problem when $v_n = n$, Pisot in [23] introduced the Borel-Laplace transform; the same idea was used later by Wallisser in the case where $v_n = q^n$; here the q-analogue $e_q(x)$ of the exponential function led to the q-Borel-Laplace transform.

In what follows, we will use the U-exponential function ([12], [13]) and the U-Laplace transform in order to solve the Newton interpolation problem. In

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Keywords and phrases: Newton interpolation series, U-Laplace transform, U-exponential function, U-exponential type.

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CONVOLUTION MULTIPLIERS ON WEIGHTED BESOV SPACES

JOSÉ LUIS ANSORENA AND OSCAR BLASCO

Abstract

We give a complete characterization of convolution multipliers between particular cases of weighted Besov classes and a description of multipliers between two weighted Besov spaces $B_w^{p,q}$ for all values of $0 < p, q \leq \infty$ in terms of multipliers between L^p spaces.

0. Introduction

The theory of Fourier multipliers on a Banach space or between two different Banach spaces defined on the torus or on Euclidean space has a long history. The first space to be studied was $L^p(\mathbb{R}^n)$, but the difficulties appearing for values of p different from $p = 1$ or $p = 2$ pushed analysts to the study of other spaces such as Hardy spaces, Bergman spaces, Besov spaces, L^p spaces with a general measure, and so on, where more information can be obtained.

Recently new results on multipliers on Banach spaces of analytic functions on the torus, such as Bergman and Besov spaces, have been achieved by several authors; see, for example, [A, B1, JJ, W]. The aim of this paper is to investigate the situation for Besov spaces defined on $\mathbb{R}^n$ and not only for potential weights but for more general ones.

The study of multipliers on Besov spaces defined on $\mathbb{R}^n$ goes back to Hardy-Littlewood, who gave a description of multipliers from a Besov space to itself in terms of multipliers on $L^p(\mathbb{R}^n)$ spaces. In [FS, P1, P2, SZ, T] the reader can find results that contain those of Hardy-Littlewood. The embeddings existing between Besov spaces and Hardy spaces make very useful the knowledge of the situation for Besov spaces to get new results in the setting of Hardy spaces (see [BaS, Jo, He]).

In this paper we get a complete description of certain cases using Herz spaces, extending previous results by R. Johnson ([Jo]) and also, by regarding Besov spaces as retracts of certain mixed norm spaces, we are able to give a complete characterisation of multipliers between Besov spaces in terms of

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QUASICONFORMAL DISTORTION OF UNIVALENT FUNCTIONS VIA HOLOMORPHIC MOTIONS

L. F. RESÉNDIS O.[1]

Abstract

In this work we study variations of univalent functions due to quasiconformal distortion. Explicit formulas for the first and second variations are obtained with the algorithm employed here.

1. Introduction

In this work we study variations of univalent (holomorphic and injective) functions which are obtained by quasiconformal distortion caused by holomorphically parametrized motions [8], [16]. Some works in this line of research are: Ahlfors [1], Sontag [17], and Yanagihara [18]. Quasiconformal distortions caused by holomorphically parametrized motions [16] have been studied in B. Rodin [10], Ch. Pommerenke, B. Rodin [10], and L. Reséndis [11].

Our main objective is to give a unified approach to different quasiconformal variations for univalent functions using holomorphic functions. In particular we recover the previous results of Ahlfors, Sontag and Yanagihara. More precisely, let $\Delta := \{\zeta \in \mathbf{C} : |\zeta| < 1\}$ and let $h\colon \Delta \to h(\Delta)$ be a univalent function. Let $\mu(\lambda)\colon \Delta \to \Delta$ be a measurable function with $|\mu(\lambda)|_\infty < 1$, depending holomorphically on λ in a neighborhood V of $0 \in \mathbf{C}$ as element of the Lebesgue space $L^\infty(\Delta)$, (in particular depending real analytically on $\operatorname{Re}\lambda$, $\operatorname{Im}\lambda$), and $\mu(0) = 0$, that is

$$\mu(\lambda)(\zeta) = \mu_1(\zeta)\lambda + \mu_2(\zeta)\lambda^2 + \cdots .$$

Also let $\nu(\lambda) : \mathbf{C} \to \Delta$ be defined by

$$\nu(\lambda)(z) = \begin{cases} \left(\mu(\lambda)\dfrac{\overline{h'}}{h'}\right) \circ h^{-1}(z) & \text{if } z \in h(\Delta) \\ 0 & \text{otherwise.} \end{cases}$$

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Keywords and phrases: Univalent functions, quasiconformal distortion, holomorphic motion.
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UPCROSSING INEQUALITIES FOR POWERS OF NONLINEAR OPERATORS AND CHACON PROCESSES

S. E. Ferrando

Abstract

In this paper we use a filling scheme technique to prove the integrability of two counting functions. The integrability of one of these functions implies the a.e.convergence for the powers $T^n f(x)/n$ where T belongs to a certain class of nonlinear operators. The other counting function generalizes the upcrossing function considered by Bishop to the case of Chacon processes. In the last section we prove the connection between our results and previous results by Bishop. We also provide a result which connects upcrossings and oscillations.

1. Introduction

References [2], [3], [4] introduced the notion of upcrossing inequalities (u.i.) in the Ergodic Theory setting. We recall the reader that the upcrossing function counts the number of times a given sequence oscillates through a given interval. Roughly speaking, these results established an integral inequality for a counting function relevant to some convergence problem. Once this inequality is established, a.e. convergence follows easily; in general there is no need to appeal to the Banach principle. Recently, new attention has been given to this phenomenon (see for example references [1], [8], [6]) and it is natural to investigate how fundamental are the counting methods implicit in the u.i. in the context of Ergodic Theory as well as in the broader context of Analysis.

In the next section of this paper we study the u.i. in the setting of nonlinear operators. The motivation behind this work is to prove that an u.i. holds for the nonlinear averages ([9], [7]) and, therefore, to find a different proof for the important new result in [10] which establishes a.e. convergence for nonlinear averages. We are able to establish a partial result in this direction, namely Theorem (2.9), which proves that the function defined as the number of times at which the powers $T^n f(x)/(n+1)$ are bigger than a certain fixed number η is integrable for a certain class of nonlinear operators T. We will indicate how this result is related to the main result in [10]. It is an open problem to extend

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CONDITIONS FOR INTEGRAL AND OTHER OPERATORS TO BE OF TRACE CLASS

RICHARD M. DUDLEY AND JOSÉ M. GONZÁLEZ-BARRIOS

Abstract

We recall that a linear operator A between Hilbert spaces is of trace class if and only if for some $c_n > 0$, for each $n = 1, 2, \dots$, there is an operator B_n of rank n such that the Schmidt norm $\|A - B_n\|_2 \le c_n$, and $\sum_n n^{-\frac{1}{2}} c_n < \infty$. From this we give a new and shorter proof of a theorem of Stinespring: If H is a Hilbert space and $k\colon \mathbb{R} \mapsto H$ is periodic of period 1, measurable, and if

$$\int_0^1 \int_0^1 \|k(x+h) - k(x)\|^2 h^{-\zeta} dh dx \ < \infty \text{ for some } \zeta > 2,$$

then the integral operator with kernel k on $L^2[0, 1]$ is of trace class. We also reobtain and extend a theorem of W. P. Kamp, R. A. Lorentz and P. A. Rejto: if K is a measurable function on the plane such that for some $\gamma > 0$,

$$\int_{\infty}^{\infty} \int_{\infty}^{\infty} K(x, y)^2 (1 + x^2)^{\gamma} dx\, dy < \infty, \quad \text{and}$$

$$\int_{\infty}^{\infty} \int_{\infty}^{\infty} (1 + x^2)^{\beta} \int_0^{\infty} h^{-1-2\alpha} [K(x+h, y) - K(x, y)]^2 dh\, dx\, dy < \infty$$

where $\frac{1}{2} < \alpha < 1$ and $\beta > \alpha + \gamma - 2\gamma\alpha$, then the integral operator with kernel K is of trace class on $L^2(\mathbb{R})$.

1. Introduction

In this section some facts about trace class operators in general will be recalled. Here the main result is that an operator A is of trace class if and only if there exist operators B_n of rank n, within c_n of A in Schmidt norm, where $\sum_n n^{-\frac{1}{2}} c_n < \infty$. Then in later sections, we reobtain and extend the sufficient

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EMBEDDINGS OF REAL PROJECTIVE SPACES

DONALD M. DAVIS

1. Statement of results

The question of finding the smallest Euclidean space in which real projective space $\mathbb{P}^n$ can be (differentiably) embedded was the subject of intense investigation during the 1960s and 1970s. The purpose of this paper is to survey the status of the question, and add a little bit to our knowledge by proving one new family of embeddings, using old methods of obstruction theory. Our new result is given in the following theorem.

THEOREM (1.1). *If $n = 2^i + 3 \geq 11$, then $\mathbb{P}^n$ can be embedded in $\mathbb{R}^{2n-4}$.*

As far as I can tell, this improves on previous embeddings by 1 dimension. Indeed, Berrick's 1979 table ([4]) lists the best embedding for $\mathbb{P}^n$ to be in $\mathbb{R}^{2n-3}$ when $n = 2^i + 3$, from [20], and I know of no embedding results for $\mathbb{P}^n$ proved subsequent to Berrick's table. (There are, however, subsequent nonembedding results, notably those of [3]).

Table 1 lists the best nonembedding and embedding results for $\mathbb{P}^n$ of which I am aware, for $n \leq 63$. Of course, most of these results fit into infinite families. Here we use the symbol $\subset$ to refer to differentiable embeddings.

Note that some of the nonembedding results (those of [1], [13], [9] and [7]) are actually obtained from nonimmersion results.

2. Proof of theorem (1.1)

In this section, we prove Theorem (1.1). Our method is that used by Mahowald in [16]. A main tool is the following result, which was proved in [16], following [8]. In Section 3, we will add a few details to the argument given in [16]. Let ξ_q denote the Hopf bundle over $\mathbb{P}^q$, and let ε denote the trivial bundle.

THEOREM (2.1). *Assume that $\mathbb{P}^q$ embeds in $\mathbb{R}^p$ with normal bundle ν. We have:*

i) If $\nu \otimes \xi_q$ has n linearly independent sections and $\mathbb{P}^{n-1}$ embeds in S^{m-1}, then $\mathbb{P}^{q+n}$ embeds in $\mathbb{R}^{p+m}$.

ii) $(\nu \otimes \xi_q) \oplus (q+1)\varepsilon \approx (p+1)\xi_q$.

Here the hypothesized embeddings need only be topological, and the embedding in the conclusion is only topological. We then use the result of Haefliger

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Keywords and phrases: embeddings, real projective spaces, obstruction theory.

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PARABOLIC UNIFORM LIMITS OF HOLOMORPHIC FLOWS IN $\mathbb{C}^2$

B. AZEVEDO SCÁRDUA

Abstract

A holomorphic vector field on a 2-dimensional Stein manifold, which is uniformly approximated by complete holomorphic vector fields, has some remarkable properties: it is complete if its orbits are parabolic. This is of special interest if we have a polynomial vector field on a complex affine space, and suggests the use of foliation-geometrical techniques in the classification of complete polynomial vector fields over $\mathbb{C}^2$.

1. Introduction

Let Z be a holomorphic vector field on a complex manifold M. Given a point $x \in M$ we denote by $\phi_x(z)$ the unique local solution of the differential equation $\dot{X} = X(z)$, $X(0) = x$. This defines a holomorphic map $\phi: V \to M$, $\phi(z, x) = \phi_x(z)$ in an open neighborhood V of $\{0\} \times M \subset \mathbb{C} \times M$, satisfying $\phi(z, \phi(w, x)) = \phi(z + w, x)$ everywhere both sides are defined. The map ϕ is called the *(local) flow* of Z on M. A point $p \in M$ is a fixed point of $t \mapsto \phi_t$ if, and only if, p is a singular point of Z, $Z(p) = 0$. Throughout this paper we will assume that Z has *isolated* singularities on M. We denote by sing(Z) this set of singular points. Fixed a point $x \in M$ we can consider the Riemann surface $\Omega_x = \Omega_x(Z)$ defined by the analytic continuation along paths in $\mathbb{C}$, of the map $z \mapsto \phi_x(z)$. As it is well known this is a *Riemann domain* over the plane $\mathbb{C}$ [8], with a natural projection denoted by $\pi_x : \Omega_x(Z) \to \mathbb{C}$ which is given by the base point projection. Moreover, the map ϕ_x separates points in the fiber Ω_x. We call $\Omega_x(Z)$ the *maximal domain* of the local flow ϕ_x of Z through x and the image $\mathcal{O}_x = \mathcal{O}_x(Z) = \pi_x(\Omega_x(Z)) \subset M$ is the *complex orbit* of Z through x. The disjoint union $\Omega = \bigcup_{x \in M} \Omega_x \times \{x\}$ has a natural structure of a complex manifold which makes the projection $\pi: \Omega \to \mathbb{C} \times M$, $\pi(z, x) = (\pi_x(z), x)$ holomorphic and endowes Ω with the structure of a holomorphic Riemann domain over $\mathbb{C} \times M$. We call this Riemann domain the *fundamental domain* of Z (of its local flow). There exists another construction of Ω which can be done if M is a

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$\bar{\partial}$-TORSION AND COMPACT ORBITS OF ANOSOV ACTIONS ON COMPLEX 3-MANIFOLDS

EUGENIO GARNICA VIGIL AND HÉCTOR SÁNCHEZ MORGADO

Abstract

In analogy with the work of Fried and Laederich we study the relation between $\bar{\partial}$-torsion of a compact complex 3-manifold M and the compact orbits of an Anosov holomorphic action on M.

1. Introduction

In 1968 Milnor [8] pointed out the remarkable similarity between the algebraic formalism of the Reidemeister torsion in topology and zeta functions à la Weil in dynamical systems theory. This theme has been thoroughly investigated by David Fried, who devised for any smooth flow and any flat bundle over the underlying manifold, a certain zeta function counting the periodic orbits of a flow with appropriate multiplicities. Fried was able to show for a variety of flows [1] that the zeta function associated to any acyclic flat bundle is actually meromorphic on a neighborhood of $[0, \infty)$, regular at 0, and that its value at 0 coincides with the Reidemeister torsion with coefficients in the given flat bundle and thus is a topological invariant. Because of the analogy with the Lefschetz fixed point formula, Fried used the term "flow with the Lefschetz property" in reference to such a flow. In particular, Fried proved that the geodesic flow of a closed manifold of constant negative curvature has the Lefschetz property [2] and we extended those results to transitive Anosov flows on 3-manifolds [10].

In analogy with their definition of analytic torsion on a Riemannian manifold, Ray and Singer define the $\bar{\partial}$-torsion for complex manifolds. Fried proved that the known connections between torsion and the dynamical features of closed orbits continue to hold in the holomorphic category [3]. He posed also a question about such connections for actions of a noncompact Lie group other than $\mathbb{R}$. Laederich [6] has investigated the case of complex manifolds which fibrate over the torus, having a one dimensional holomorphic foliation transverse to the fibers. He found a formula relating $\bar{\partial}$-torsion to "theta" functions associated to the compact orbits of the foliation.

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COMPORTAMIENTO ASINTÓTICO PERIÓDICO DE LAS MATRICES DE MARKOV

CÉSAR E. VILLARREAL

Resumen

Se halla en forma explícita la descomposición periódica asintótica de una matriz de Markov $n \times n$ vista como un operador de Markov sobre $\mathbb{R}^n$. Además, se describe el conjunto de sus distribuciones estacionarias.

Abstract

We give explicitely the asymptotic periodic decomposition of a $n \times n$ Markov matrix as a Markov operator on $\mathbb{R}^n$. We describe also, the set of its stationary distributions.

1. Introducción

En este trabajo se estudia el comportamiento asintótico para matrices de Markov P de orden n, vistas como operadores de Markov sobre $\mathbb{R}^n$. Presentamos dos resultados principales para tales matrices. El primero, Teorema (3.1), es el Teorema de Descomposición Espectral (TDE) para una matriz de Markov P, el cual determina la descomposición periódica asintótica de P. Nuestro resultado es un caso particular de versiones más generales del TDE (ver [2,3,4]), pero el hecho de restringirnos al caso de matrices (operadores) de Markov de orden finito nos permite obtener una caracterización explícita y una manera de calcular el conjunto D_∞ de las distribuciones periódicas asociadas a P. Nuestro segundo resultado principal, el Teorema (4.1), presenta una caracterización del conjunto $D^*_\infty \subset D_\infty$ de las distribuciones estacionarias de P. Estos resultados requieren, en particular, un análisis detallado de las matrices de Markov idempotentes, el cual se presenta en la sección 2.

Nuestro punto de partida es el artículo de Chi [1] que estudia el conjunto D_∞ de los puntos límites de sucesiones $\{P^m(\mu)\}_{m=1}^\infty$ con $\mu = (x_1, x_2, \ldots, x_n)^T$, $x_i \geq 0$, y $\sum_{i=1}^n x_i = 1$. Con el objeto de describir en forma más precisa nuestros resultados a continuación presentamos algunos conceptos relacionados.

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