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DYNAMIQUE DES GROUPS D'AUTOMORPHISMES DE $\mathbb{C}, 0$

FRANK LORAY

Abstract

We present a short survey about the dynamics of finitely generated subgroups of Aut($\mathbb{C}$, 0). In fact, we sum-up the results which seem essential for us into two theorems which are the consequence of an amount of works over the last century on the subject. The first one asserts that solvable subgroups act, in general, like the affine group of $\mathbb{C}$ at a neighborhood of infinity. The second one shows that the dynamics are systematicaly chaotic (density of orbits, of fixed points, ...) as soon as the subgroup is non solvable.

Résumé

Nous présentons un bref survol de la dynamique des sous-groupes de type fini de Aut($\mathbb{C}$, 0). En fait, nous résumons les résultats qui nous semblent essentiels sous la forme deux théorèmes qui sont le fruit d'un siècle de travaux sur le sujet. Le premier nous dit que les sous-groupes résolubles agissent, en général, comme le groupe affine sur $\mathbb{C}$ au voisinage de l'infini. Le second montre qu'*a contrario* la dynamique est systématiquement chaotique (densité des orbites, des points fixes, etc...) dès lors que le sous-groupe n'est pas résoluble. L'originalité de cet exposé vient peut-être de la simplicité et de l'efficacité avec laquelle nous avons tenté de présenter résultats et démonstrations.

0. Introduction

On note Aut($\mathbb{C}$, 0) le groupe pour la composition des germes de transformations conformes localement inversibles de $\mathbb{C}$ fixant 0 :

$$\mathrm{Aut}(\mathbb{C},0) = \{f(z) = az + \dots \ ; \ f \in \mathbb{C}\{z\} \text{ et } a \neq 0\}.$$

On définit la dynamique d'un sous-groupe $\widehat{G}$ de type fini de Aut($\mathbb{C}$, 0) comme suit. On se donne un système de générateurs $f_1, \dots, f_p \in \mathrm{Aut}(\mathbb{C}, 0)$ de $\widehat{G}$ et un voisinage U de $0 \in \mathbb{C}$ sur lequel les générateurs f_i et leurs inverses $f_{i+p} = f_i^{\circ(-1)}$ sont bien définis injectifs pour $i = 1, \dots, p$. La dynamique qui nous intéresse est celle du pseudo-groupe G engendré sur U par les transformations

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Keywords and phrases: pseudo-groupes conformes, points fixes, densité des orbites, feuilletages holomorphes et holonomie.

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THE PSEUDO-ARC

WAYNE LEWIS

Abstract

The *pseudo-arc* is the simplest nondegenerate hereditarily indecomposable continuum. It is, however, also the most important, being homogeneous, having several characterizations, and having a variety of useful mapping properties.

The pseudo-arc has appeared in many areas of continuum theory, as well as in several topics in geometric topology, and is beginning to make its appearance in dynamical systems.

In this monograph, we give a survey of basic results and examples involving the pseudo-arc. A more complete treatment will be given in a book [133] dedicated to this topic, currently under preparation by this author.

We omit formal proofs from this presentation, but do try to give indications of some basic arguments and construction techniques.

Our presentation covers the following major topics.

1. Construction
2. Homogeneity
3. Characterizations
4. Mapping properties
5. Hyperspaces
6. Homeomorphism groups
7. Continuous decompositions
8. Dynamics

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Keywords and phrases: pseudo-arc, indecomposable continuum, homogeneous, hyperspace, continuous decomposition, weakly chainable.

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PARITY OF MOD p BETTI NUMBERS

AVNER ASH

Abstract

We construct nondegenerate pairings in the "middle dimension" on parts of the mod-p cohomology of a p-manifold with boundary of odd dimension n. When $n \equiv 1 \pmod 4$, the pairings are alternating. As a corollary we explain the experimental results of [AM1] that if N is an odd prime, $N \leq 223$, and $5 \leq p \leq 23$ then the mod p quasi-cuspidal homology in degree 3 of $\Gamma_0(N)$ of $GL(3, \mathbb{Z})$ is even dimensional if $N \not\equiv 1 \pmod p$, and odd dimensional otherwise.

1. Introduction

Let p be a prime, m a positive integer, and let M denote a compact $2m+1$-dimensional p-manifold with (possibly empty) boundary. This means that locally M is a quotient of a manifold (or a manifold with boundary) by a group which acts with finite stabilizers with orders prime to p. We define the *interior* cohomology of M to be the image of $H^k(M, \partial M)$ in $H^k(M)$, and we denote it by $H^k_!(M)$.

In this paper we prove the existence of a natural filtration on the m-dimensional mod p interior cohomology of M with trivial coefficients such that the successive quotients (except for the first) support a naturally defined nondegenerate bilinear form. This form is symmetric if m is odd and alternating if m is even. Hence in the latter case these quotients are even dimensional if p is odd. We do this under the hypothesis that there is no p-torsion in the $(m-1)$-st integral homology of the boundary of M.

Our construction uses the Bockstein spectral sequence and Lefschetz duality. Similar results for ordinary cohomology appear in [R] and [B] for manifolds without boundary, and in [W] for manifolds with boundary, assuming that the homology of the boundary in dimensions m and $m+1$ vanishes.

The motivation of this paper was to explain the experimental findings of [AM1] concerning the parity of the dimension of the quasi-cuspidal mod p homology of $\Gamma_0(N)$ in $GL(3, \mathbb{Z})$. Here N is an odd prime and $\Gamma_0(N)$ is the subgroup

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Keywords and phrases: Bockstein, cohomology, arithmetic group.

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A CHARACTERIZATION OF POSITIVE UNIT FORMS

M. BAROT

Abstract

This paper considers unit forms, i.e. positive definite quadratic forms with unitary coefficients in the quadratic terms. The classes of equivalence of connected unit forms are given by Dynkin diagrams. A characterization is presented by the associated bigraphs of positive definite unit forms which are equivalent to $\mathbb{A}_n$ for some integer n.

1. Introduction and results

An integral quadratic form

$$q : \mathbb{Z}^n \to \mathbb{Z}, \quad q(x) = \sum_{i=1}^{n} q_i x(i)^2 + \sum_{i<j} q_{ij} x(i)x(j)$$

is called *unit form* provided $q_i = 1$ for all i. Unit forms play an important role in the theory of representations of algebras as associated forms to a finite dimensional algebra over an algebraically closed field: the Tits form and in case the algebra has finite global dimension also the Euler form. Their properties, such as (weak) positivity or (weak) non-negativity, reflect properties of the algebras, see for example [4, 5, 2, 3].

The form is *positive* if $q(x) > 0$ for all non-zero $x \in \mathbb{Z}^n$. Clearly, for a positive unit form q we must have $|q_{ij}| \leq 1$ for all $i < j$. For convenience we set $q_{ji} = q_{ij}$ for $i < j$. To a unit form $q : \mathbb{Z}^n \to \mathbb{Z}$ we associate a bigraph $\mathrm{B}(q)$ with vertices $1, \ldots, n$ and edges as follows. Two different vertices i and j are joined by $-q_{ij}$ *full* edges if $q_{ij} \leq 0$ and by q_{ij} *broken* edges if $q_{ij} > 0$. Clearly, any *reduced* bigraph (that is, between two vertices i and j there are not both full and broken edges) Γ without loops is isomorphic to $\mathrm{B}(q)$ for some unit form q, which we denote by $\mathrm{q}(\Gamma)$. A unit form q is *connected* if so is $\mathrm{B}(q)$. In the following we assume that bigraphs are reduced and without loop, furthermore, for a bigraph B, we define $[i, j]_B = -\mathrm{q}(B)_{ij}$.

The *frame* of a bigraph B is the graph $\Phi(B)$ which is obtained from B by turning the broken edges into full ones.

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Keywords and phrases: unit form, Dynkin-diagram.

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DEFORMATIONS OF COMPLEX HYPERSURFACES ON G/P AND AN INFINITESIMAL TORELLI THEOREM

PEDRO L. DEL ANGEL R.

Abstract

This paper addresses the problem whether the family of complex hypersurfaces of a given multidegree on a generalized flag manifold ($\mathbf{G}/\mathbf{P}$) is a locally complete deformation or not. An explicit solution to this problem is given in theorem (3.2) for the classical groups, the special groups are considered independently in theorems (5.2) and (6.1). The question was previously considered by J. Wehler on [5] for hypersurfaces in a manifold of complete flags (that is, $\mathbf{G} = \mathbf{S}L_n$ and $\mathbf{P} = \mathbf{B}$ a Borel subgroup of $\mathbf{G}$). We will actually follow the line of [5] and this reduces the question to the vanishing of the group $H^2(\mathbf{G}/\mathbf{P}, \tau(-d))$.

By the same token one can give conditions under which the infinitesimal Torelli theorem holds for every smooth hypersurface of given multidegree in a generalized flag manifold. Here we use a criterion developed by Flenner in [3] to reduce the question to the vanishing of certain cohomology groups and the surjectivity of certain maps (see theorem (4.2)). It is done, for the classical gropus, in theorem (4.2), and for the special groups in theorems (5.4) and (6.3).

This article is divided in two parts, part 1 (especially sections 2-4) consider only the case of classical groups, while the special groups (except for G_2, where our method only work for the case of the Borel subgroup) are treated in part two (sections 5 and 6).

In section 0 we introduce the necessary notations and some well known facts about generalized flag manifolds for groups of classical type.

In section 1 we state auxiliary combinatoric results.

In section 2 we prove a vanishing theorem for the groups $H^k(\mathbf{G}/\mathbf{P}, \tau(-d))$ if $\mathbf{G}$ is of type A_l, B_l, C_l or D_l.

Section 3 is devoted to the problem of deformations of hypersurfaces, and section 4 considers the question of the infinitesimal Torelli theorem for those groups.

Section 5 deals with the case of groups of type E_6, E_7 and E_8, while the groups of type F_4 are considered in section 6.

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ON CURVES AND ORBITS OF MATRICES

A. G. RAGGI-CÁRDENAS AND L. SALMERÓN

Abstract

Given an algebraically closed field k, we consider the action of the general linear group G over the space $k^{n\times n}$, of $n\times n$ matrices, given by conjugation. We prove the following geometric fact: Let $\ell\colon k \longrightarrow k^{n\times n}$ be any curve which does not lie in some G-orbit, then there is a curve $\widehat{\ell}$ which hits exactly the same orbits as ℓ does (with only finitely many possible exceptions) and satisfies the additional property $\widehat{\ell}(t)$, $\widehat{\ell}(t')$ are not conjugate for all pairs of different points t, t' in a cofinite subset of k.

1. Introduction

(1.1). Throughout this note k denotes an algebraically closed field. Let $k^{n\times n}$ denote the afine space of $n\times n$ matrices over k. Then the general linear group $G = G_n$ acts on $k^{n\times n}$ by conjugation. We write $M \cong N$ whenever the matrices M and N belong to the same orbit under this action. By definition a *curve* (*line*) ℓ in $k^{n\times n}$ is a map $\ell\colon k \longrightarrow k^{n\times n}$ such that all entries of $\ell(t)$ are described by polynomials in t (resp. of the form $\ell(t) = tM + N$, where M, N are fixed matrices in $k^{n\times n}$). Abusing the language, we shall say that a curve ℓ *lies in* a constructible subset $\mathscr{H}$ of $k^{n\times n}$, if $\ell(t) \in \mathscr{H}$ for almost all $t \in k$. A curve ℓ is called *degenerate* if it lies in one orbit of $k^{n\times n}$. It is called *reliable* if $\ell(t) \ncong \ell(t_1)$, for all pairs of different points t, t_1 in a cofinite subset of k. Finally, we say that the curve $\widehat{\ell}$ in $k^{n\times n}$ *reparametrizes* the non-degenerate curve ℓ in $k^{n\times n}$ if, for almost all $t \in k$, there is a $\widehat{t} \in k$ with $\widehat{\ell}(\widehat{t}) \cong \ell(t)$.

Given a curve ℓ in $k^{n\times n}$, we denote the set $G\,\mathrm{Im}(\ell)$ simply by $G\ell$. Thus the curve $\widehat{\ell}$ reparametrizes the non-degenerate curve ℓ iff ℓ lies in $G\widehat{\ell}$. Naturally, in this case, we call $\widehat{\ell}$ a *reparametrization of* ℓ. With all this notation in mind, the following is easy to show.

Remarks (1.2). If the curve $\widehat{\ell}$ reparametrizes the non-degenerate curve ℓ, then $\widehat{\ell}$ is non-degenerate. Moreover, in the class of non-degenerate curves in

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Keywords and phrases: family, moduli space, stratification, vector spaces, endomorphisms, canonical forms.

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MAXIMAL RESOLVABILITY OF SOME TOPOLOGICAL SPACES

LUIS MIGUEL VILLEGAS SILVA

Abstract

We use several methods to show that some classes of topological spaces (e.g. Fréchet-Urysohn, biradial, bisequential, free groups and κ-bounded groups) are maximal resolvable. We also obtain such decompositions for certain product spaces.

1. Introduction

This paper deals with the problem of finding conditions for maximal or at least large resolvability of topological spaces. The concept of resolvability was introduced by Hewitt [H] and also by Katětov. We use the idea of Katětov in order to explain the origin of the problem.

In 1947 Katětov [K1] posed the following problem: Is there a topological space X without isolated points which has the property that every real-valued function defined on X is continuous at some point? Another related question was formulated in [M1]: Does there exist a dense in itself topological space X such that every real-valued function defined on X is continuous on some open dense subspace of X?

Consider a space X which contains two disjoint dense subspaces E and D such that $X = D \cup E$. Define a function $f: X \to \mathbb{R}$ by $f(E) = 0$ and $f(D) = 1$. It is obvious that f cannot be continuous at any point of X.

So we must look for the positive answer of Katetov's question in the class of irresolvable spaces, i.e., those spaces which contain no disjoint dense sets.

In this paper we are interested on finding a decomposition of a space into a union of large families of disjoint dense sets. All of our results go in this direction. Section 2 presents the necessary notation, definitions and results which form the basic preliminary material for the rest of the paper. In Section 3 we discuss the resolvability of spaces satisfying some kind of convergence conditions. Every κ-bounded topological group of large cardinality is maximal resolvable as is shown in Section 5 (see remarks at the beginning of that

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Keywords and phrases: resolvability, κ-bounded group, network, biradial space, free topological group, bisequential space, maximal resolvability.

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A NOTE ON CHEBYSHEV'S 'OTHER' INEQUALITY

JOSÉ A. CANAVATI AND FERNANDO GALAZ-FONTES

Abstract

Here we show that Chebyshev's inequality $\int_a^b d\mu \int_a^b f(x)g(x)\,d\mu \geq \int_a^b f(x)\,d\mu$ $\int_a^b g(x)\,d\mu$, can be generalized into several different contexts, by means of elementary measure theoretical arguments.

1. Introduction

Let μ be a probability measure on the real line and $f(x)$ and $g(x)$ increasing functions. Then

$$\int_{-\infty}^{\infty} f(x)g(x)\,d\mu \geq \int_{-\infty}^{\infty} f(x)\,d\mu \int_{-\infty}^{\infty} g(x)\,d\mu$$

says that the random variables $f(x)$ and $g(x)$ are positively correlated. This is Chebyshev's 'other' inequality (cf. [1],[2],[4]).

In this note we would like to explore more general settings in which this inequality holds. For this we start with the following result originally observed by Andreief (1883) (cf. [2]) for the case of the finite interval $[a, b]$ and a positive measure μ is given by an integral: $\mu([s,t]) = \int_s^t \sigma(t)\,dt$ $(a \leq s \leq t \leq b)$, where σ is a nonnegative integrable function on $[a, b]$.

CHEBYSHEV'S 'OTHER' INEQUALITY. *If $f(x)$ and $g(x)$ are continuous functions on $[a, b]$ satisfying*

$$(*) \qquad \left[f(x) - f(y)\right]\left[g(x) - g(y)\right] \geq 0 \quad \textit{for all} \quad x, y \in [a, b],$$

then

$$(**) \qquad \int_a^b d\mu \int_a^b f(x)g(x)\,d\mu \geq \int_a^b f(x)\,d\mu \int_a^b g(x)\,d\mu\,.$$

The condition $(*)$ is read "$f(x)$ and $g(x)$ are similarly ordered" (cf. [3]).

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Keywords and phrases: Chebyshev inequality, topological group.

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PERTURBED ZEROS OF CLASSICAL ORTHOGONAL POLYNOMIALS

R. G. CAMPOS

Abstract

It is well known that the set of N zeros of polynomials satisfying linear homogeneous second order differential equations is the solution of certain system of N nonlinear equations depending on a function related to one of the coefficients of the differential equation and, in the case of the classical orthogonal polynomials, to the corresponding weight function. By perturbing such a function we generate sets of perturbed zeros. This procedure yields a linear problem for the differences between the original and perturbed zeros. The matrix of this linear problem is the one used by Stieltjes to show the monotonic variation of the zeros of the classical orthogonal polynomials. The elementary method presented here can be used to give a bound for the error produced in approximating the original set of zeros by the perturbed one. As simple examples we obtain a bound for the euclidean norm of the vector of differences between the zeros of Hermite polynomials and those of (appropriately scaled) Gegenbauer and factorial polynomials.

1. An approximation problem

Let f be a polynomial of degree N satisfying the second order differential equation

$$f''(t) + a_1(t)f'(t) + a_0(t)f(t) = 0\,. \tag{1.1}$$

It is well known that the N zeros of f, $x_1, x_2, \ldots, x_n$, satisfy the nonlinear equations

$$\sum_{l=1}^{N}{}' \frac{1}{x_j - x_l} = -\frac{\gamma'(x_j)}{\gamma(x_j)} \qquad j = 1, 2, \ldots, N, \tag{1.2}$$

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Keywords and phrases: classical orthogonal polynomials, zeros, Stieltjes matrix.

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ON THE POINCARÉ-LYAPUNOV CENTRE THEOREM

MARCO BRUNELLA AND MASSIMO VILLARINI

Abstract

We give a geometric proof and a slight generalization of a result of Sibuya and Urabe concerning a higher dimensional version of the classical Poincaré - Lyapunov Centre Theorem. This is done from a complexified point of view.

1. Introduction

The classical Poincaré-Lyapunov Centre Theorem says that a centre-type singularity of an analytic vector field on the plane has an analytic first integral, provided that the linear part of the vector field at the singular point generates a (nontrivial) rotation. Equivalently, up to a multiplication by a nonvanishing function, the vector field is linearizable near the nondegenerate centre. A geometric proof of this result was given by Moussu [4]: his argument motivated part of this article.

We consider the multidimensional analog of the planar centre dynamics: we will call it *multicentre*. We will give, in the same spirit of [4], a geometric proof of a result of Urabe and Sibuya [6], stating that an analytic multicentre, with linear part at the singular point which generates a multirotation (see definitions in the next section), is always linearizable. Moreover we will generalize the quoted result by Urabe and Sibuya. The main tools in the proof of these results are the analytic extension up to the origin of the period function, obtained through the analysis of the complexification of the vector field, and the use of a theorem by Bochner [3] on the linearization of compact group actions in a neighbourhood of a fixed point. As a byproduct of these arguments we obtain still another proof of Poincaré - Lyapunov Centre Theorem.

2. Linearization of multicentres

An isolated singular point O of a planar vector field is a *centre* if there exists a neighbourhood U of O, such that $U \setminus \{O\}$ is filled by closed nontrivial trajectories. A centre at O is *nondegenerate* if the linear part at O of the vector field has eigenvalues $\pm i\omega$, $\omega > 0$. The classical Poincaré - Lyapunov Centre Theorem [5], [2] states

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Keywords and phrases: singularities of vector fields, centre-type singularity, linearization, complexification, stability, periodic orbits.

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GEOMETRIC INVARIANTS OF SMOOTH LOW DIMENSIONAL DYNAMICAL SYSTEMS

FRANCISCO SOLIS

Abstract

The determination of invariants of lower dimensional dynamical systems that are captured by local adaptive Galerkin bases is achieved. We explicitly compute these invariants for the cases of smooth curves and smooth surfaces in $\mathbb{R}^3$. We show that these invariants contain geometrical information, such as dimension and extrinsic information. We discover that the coordinate system associated with these bases is given by a generalization of a canonical system, namely the Frenet frame.

1. Introduction

Often when one studies a dynamical system the interest is in the behavior of the orbits after long periods of time. Many systems exhibit transient behavior followed by an asymptotic motion lying on a subset of the phase space, an attracting set. More importantly, this subset is contained in some finite dimensional manifold. One can try to understand the different levels of complexity of the orbits by analyzing the geometric structure of the attracting sets.

Two basic properties of these attracting sets that we will consider are its dimension and its local geometric shape. To study these properties we reconsider the idea, from [1], of embedding the attracting set, which we assume to be a smooth manifold, in a set of local patches and finding in each patch a coordinate system which is optimal in the sense that the error of the projection of the orbits on this system is minimal.

Assume that there is a dynamical system acting on a separable Hilbert space H with an attractor X which has an invariant measure μ. From now on, we assume that X is a manifold with $\operatorname{supp}\mu = X$.

Consider any orthonormal basis $\{b_i\}_{i=1}^{\infty}$. For any $h \in H$ we define the error of projecting the orbit of h, $\phi(h,t)$, into the first k basis elements as:

$$e_k(h,t) \equiv \phi(h,t) - \sum_{i=1}^{k} < \phi(h,t), b_i > b_i \, .$$

1991 *Mathematics Subject Classification*: 34A26, 58F12.

Keywords and phrases: osculating spaces, curvature, torsion.

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A UNIFORM BOUNDEDNESS PRINCIPLE FOR SPACES WITH A FAMILY OF PROJECTIONS

CHARLES SWARTZ

Abstract

We establish a uniform boundedness principle for operators defined on a topological vector space which is equipped with a family of projections satisfying a gliding hump property and with values in a semi-convex topological vector space. Applications to spaces of vector-valued functions are given.

In [Sw2], [Sw3], we established a uniform boundedness principle for locally convex spaces which are equipped with a family of projection operators satisfying a gliding hump and a decomposition property. Earlier similar uniform boundedness results for locally convex spaces with a family of projection operators were established by Drewnowski, Florencio and Paúl and used to show that the space of Pettis integrable functions, while not usually complete, is always barrelled ([DFP1], [DFP2]); see also [DFFP] for further similar results. In this note we show that the methods of [Sw2], [Sw3], based on the Antosik-Mikusinski Matrix Theorem, can be used to establish a uniform boundedness principle for non-locally convex spaces and show that in the locally convex space case the spaces have a property stronger than being barrelled.

We begin by fixing the notation and terminology which will be employed in the sequel. Let E be a Hausdorff topological vector space and let $\mathcal{A}$ be an algebra of subsets of a set S. We assume that there exists a mapping $P: \mathcal{A} \to L(E)$, the space of continuous linear operators from E into E. For $A \in \mathcal{A}$ we write $P(A) = P_A$ and assume that $P_\emptyset = 0$, $P_S = I$, the identity operator on E, and that P is finitely additive. In order to establish our uniform boundedness principle we impose two further conditions on the mapping P. We first describe the gliding hump property which will be imposed.

We say that P satisfies the strong gliding hump property (SGHP) if: $x_k \to 0$ in E and $\{A_k\} \subset \mathcal{A}$ pairwise disjoint implies there exist an increasing sequence $\{n_k\}$ and $x \in E$ such that $\sum_{k=1}^{\infty} P_{A_{n_k}} x_{n_k} = x$, where the series converges in E.

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Keywords and phrases: uniform boundedness, gliding hump, Banach-Steinhaus.

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THE CANONICAL BUNDLE OF A HERMITIAN MANIFOLD

GIL BOR AND LUIS HERNÁNDEZ-LAMONEDA

Abstract

This note contains a simple formula (Proposition (3.5) in Section 3) for the curvature of the canonical line bundle on a hermitian manifold, using the Levi-Civita connection (instead of the more usual hermitian connection, compatible with the holomorphic structure). As an immediate application of this formula we derive the following: *the six-sphere does not admit a complex structure, orthogonal with respect to any metric in a neighborhood of the round one.* Moreover, we obtain such a neighborhood in terms of explicit bounds on the eigenvalues of the curvature operator. This extends a theorem of LeBrun.

1. Introduction

First, some standard definitions. An *almost-complex structure* on an even-dimensional manifold M^{2n} is a smooth endomorphism $J: TM \to TM$, such that $J^2 = -Id$. The standard example is $M = \mathbb{C}^n$ with J given by the usual scalar multiplication by i. A *holomorphic map* between two almost-complex manifolds (M_1, J_1) and (M_2, J_2) is a smooth map $f: M_1 \to M_2$ satisfying $df \circ J_1 = J_2 \circ df$. An almost-complex structure is said to be *integrable,* or is called simply a *complex structure,* if it is locally holomorphicaly diffeomorphic to the standard example; in other words, for each $x \in M$ there exist neighbourhoods $U \subset M$, $x \in U$, and $V \subset \mathbb{C}^n$, and a holomorphic diffeomorphism $f: U \to V$.

Given an even-dimensional manifold, how is one to decide if it admits a complex structure? There are some, more or less obvious, necessary conditions (e.g. the existence of an almost-complex structure, which can be tested by characteristic classes), but in general there is no known answer to this question. A well-known example, so far undecided, is the 6-sphere (this is the only interesting dimension, because in all other dimensions $n \neq 2, 6$, the n-sphere does not admit even an almost-complex structure). This space admits a non-integrable almost-complex structure, but it is unknown as yet if it admits a complex structure.

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Keywords and phrases: Hermitian manifold, almost-complex structure, canonical bundle, curvature.

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UNIT ROOT TEST: AN UNCONDITIONAL MAXIMUM LIKELIHOOD APPROACH

GRACIELA GONZÁLEZ-FARÍAS AND DAVID A. DICKEY

Abstract

We investigate a test for unit roots in autoregressive time series based on the maximization of the unconditional likelihood. Models including mean and time trend adjustments are considered. We give percentiles for the resulting tests which are more powerful than the currently popular least squares tests. We show that the limit distributions are unchanged for higher order models so that the tests can be used in models with more than one lag. Both normalized bias tests and studentized tests are considered.

1. Introduction

Time series modeling often involves the selection and fitting of an ARIMA (autoregressive integrated moving average) model. The order of integration is defined as the degree of differencing required to make the series stationary where stationarity implies constant mean and variance over time and a covariance which depends only on the time separating two observations. The fitting of a series traditionally involves differencing the data if necessary, until they appear stationary then fitting autoregressive and moving average parameters to the (possible differenced) data. We investigate statistical ways to check whether differencing is necessary.

Appropriate differencing renders a series stationary and thus makes the resulting estimation theory easier to work out. The results tend to be classical in nature, for example normal limit distribution of estimators. Classic methods of estimation, such as least squares and maximum likelihood are not necessarily poor estimation methods for the nonstationary series, however the distributions are not standard even in the limit. If percentiles of the distributions can be obtained, then these can be used for hypothesis testing.

For ARIMA models, stationarity can be characterized by a condition on the roots of a polynomial involving the autoregressive coefficients, called the characteristic polynomial. If all the roots are larger than 1 in magnitude, the series is stationary. Therefore we can base a test for stationarity on the coefficients or roots of the characteristic polynomial. These in turn must be estimated in

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