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ALGEBRAIC CYCLES AND ANTIHOLOMORPHIC INVOLUTIONS ON PROJECTIVE SPACES

JACOB MOSTOVOY

ABSTRACT. We study the topology of the groups of real and quaternionic algebraic cycles on complex projective spaces. By "quaternionic" cycles here we mean those invariant under the generalized antipodal map on $\mathbf{CP}^{2n+1}$.

1. Introduction

The first step in the development of what is known now as "Lawson homology" was H.B. Lawson's proof of the algebraic version of the Thom isomorphism for a certain class of bundles [9] and, as a consequence, a complete description of the groups of algebraic cycles on $\mathbf{CP}^n$ from the topological point of view.

In this paper we describe two real counterparts of the results of [9]. Usually, the term "real variety" means "complex variety with an antiholomorphic involution on it". In concrete examples, however, reality comes in very different flavours. On the Riemann sphere, for example, there are two essentially different antiholomorphic involutions: complex conjugation and the antipodal map. The analogue of the antipodal map exists on any odd-dimensional complex projective space; the "reality" associated with it has a close relationship to quaternions.

Taking this into account, by "real algebraic cycles" in $\mathbf{CP}^n$ we will mean those invariant under complex conjugation. Cycles that are invariant under the generalised antipodal map will be called "quaternionic cycles".

Our main results are the calculations of the homotopy groups of the groups of real and quaternionic cycles on complex projective spaces. The real case has already been treated by T.K. Lam [8], who calculated the homotopy type of what he called "groups of mod 2 cycles". We complement his calculation by computing the homotopy type of the groups of cycles with integral coefficients. (In a recent work [10] by H.B. Lawson, P.Lima-Filho and M.-L. Michelsohn the authors also calculate the homotopy type of these groups. The results of [10] are more complete than ours as they include a description of the multiplicative structure on the homotopy of the groups of real cycles.)

As for quaternionic cycle groups, we determine their homotopy type only rationally. Stronger statements about groups of quaternionic cycles of even dimension were obtained by Lawson, Lima-Filho and Michelsohn in [11], but the rational description of odd-dimensional cycle groups seems to be the best available at the moment.

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SPLITTING THE AUTOMORPHISM GROUP OF AN ABELIAN p-GROUP

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ABSTRACT. Let G be an abelian p-group sum of finite homocyclic groups G_i. Here, we determine in which cases the automorphism group of G splits over $\ker\sigma$, where $\sigma : \mathrm{Aut}(G) \to \prod_i \mathrm{Aut}(G_i/pG_i)$ is the natural epimorphism.

1. Preliminaries

Throughout this paper, p is an arbitrary prime and r is a fixed ordinal number. Let $G = \oplus_{i \leq r} G_i$ be an abelian p-group, such that G_i is an homocyclic group of exponent p^{n_i} and finite p-rank r_i, with $n_i < n_{i+1}$ for all i. It is known that $\mathcal{E}$ the endomorphism ring of G is isomorphic to the ring $E(G)$ of all row finite $r \times r$-matrices (A_{ij}) where $A_{ij} \in Hom(G_i, G_j)$. We denote by $\mathcal{A}(H)$, the automorphism group of a group H and consider $\mathcal{A}(H)$ as the group of units of the endomorphism ring $E(H)$.

Let σ be the natural epimorphism of $\mathcal{A}(G)$ onto the product of the $\mathcal{A}(G_i/pG_i)$. We have the following exact sequence (see [2], page 256)

$$1 \to \ker\sigma \to \mathcal{A}(G) \to \prod_i \mathcal{A}(G_i/pG_i) \cong \prod_i \mathrm{GL}_{r_i}(\mathbf{Z}_p) \to 1, \tag{1.1}$$

where $\mathrm{GL}_{r_i}(\mathbf{Z}_p)$ is the general linear group of $r_i \times r_i$-matrices over the field $\mathbf{Z}_p$.

In this paper we prove Theorem (2.1) which together with Theorem (1.1) proved in [1] and [3], give a necessary and sufficient condition for the decomposition of $\mathcal{A}(G)$ as a semidirect product of $\ker\sigma = \Delta(G)$ by $\Pi(G) = \prod_i \mathrm{GL}_{r_i}(\mathbf{Z}_p)$, whenever $p \geq 5$. For the cases $p = 2, 3$ we give sufficient and necessary conditions for such decomposition in case $n_i + 1 < n_{i+1}$ for all i.

Because the p-rank of G_i is finite, $\mathrm{Hom}(G_i, G_j) \cong p^{n_j - n_i} M_{r_i \times r_j}(\mathbf{Z}_{p^{n_i}})$ for $i < j$, where $M_{r_i \times r_j}(\mathbf{Z}_{p^{n_i}})$ denotes the additive group of $r_i \times r_j$-matrices over the integers modulo p^{n_i}.

Evidently $\Delta(G) = 1 + I$, where

$$I = \{(A_{ij})_{r\times r} \in E(G) | A_{ii} \equiv 0 (\mathrm{mod}\, p), \text{ for all } i\}.$$

If r is a natural number, then G is finite and because $\mathrm{GL}_{r_i}(\mathbf{Z}_p)$ does not have any normal p-subgroup, $\Delta(G)$ is the maximal normal p-subgroup of $\mathcal{A}(G)$, denoted $O_p(\mathcal{A}(G))$.

Consider the exact sequences

$$1 \to \ker\lambda_i \to \mathcal{A}(G_i) \to \mathrm{GL}_{r_i}(\mathbf{Z}_p) \to 1, \tag{1.2$_i$}$$

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THE RANGE OF A SYSTEM OF FUNCTIONALS FOR THE MONTEL UNIVALENT FUNCTIONS

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Abstract. The paper is devoted to applications of the method of moduli of families of curves to estimation of the range of the system of functionals $(|f(z)|, |f'(z)|)$, $-1 < z < 0$ in the class of univalent maps from the unit disk with two preassigned positive real values $0, \omega \in (0,1)$.

1. Introduction

Denote by U the unit disk $\{z : |z| < 1\}$ in the complex plane $\mathbb{C}$. We consider the class $\mathcal{M}(\omega)$ of all univalent holomorphic maps $f : U \to \mathbb{C}$ normalized by the conditions $f(0) = 0$ and $f(\omega) = \omega$, where $\omega \in (0,1)$. This class of functions is known as the class of the Montel functions (see e.g., [3–6, 9]). We mention here the systematic treatment of these functions made by J. Krzyż, E. J. Zlotkiewicz, R. J. Libera [3–6, 9, 11]. In particular, for the class $\mathcal{M}(\omega)$ J. Krzyż, E. J. Zlotkiewicz [6] have found the Koebe set, i.e. $\bigcap_{f\in\mathcal{M}(\omega)} f(U)$, J. Krzyż [3–5] in the series of papers has defined the set of values of $f(z)$ for z fixed in U by the variational method. This study was continued by M. O. Reade, E. J. Zlotkiewicz [11]. Recently, the study of the bounded Montel functions has been started by R. J. Libera, E. J. Zlotkiewicz [9] and by the authors [14, 15]. The distortion theorems for the class S of all univalent holomorphic functions with the normalization $f(0) = 0$, $f'(0) = 1$ are well-known as well as estimates of systems of functionals depending on the derivative of f. The rotation in the class S helps us to derive them but for the classes $\mathcal{M}(\omega)$ and its subclass of bounded functions. In [13] we have found some sharp estimates of functionals connected with the distortion under the Montel functions. Namely, for $z = r$

$$|f'(r)| \le \frac{(1-\omega)^2(1+r)}{(1-r)(1-\omega r)(r-\omega)}|f(r)-\omega|, \quad \text{for } \omega < r < 1,$$

with the extremal function

$$f^*(z) = \frac{(1-\omega)^2 z}{(1-z)^2},$$

and

$$|f'(r)| \le \frac{1-r^2}{r(1-\omega r)(\omega - r)}|f(r)-\omega||f(r)|, \quad \text{for } 0 < r < \omega,$$

with the extremal function

$$f^*(z) = \frac{(1+\omega)^2 z}{(1+z)^2}.$$

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ON A MELLIN TRANSFORM OF THE GENERALIZED HERMITE POLYNOMIALS

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ABSTRACT. We derive Mellin transforms for the generalized Hermite polynomials $H_n^{(\mu)}(x)$ and for a corresponding extension of the exponential function $\exp(x)$.

Szegö introduced in [1] an orthogonal polynomial system $\{H_n^{(\mu)}(x)\}$, defined as

$$
\begin{aligned}
H_{2n}^{(\mu)}(x) &= (-1)^n\, 2^{2n}\, n!\, L_n^{(\mu-1/2)}(x^2),\\
H_{2n+1}^{(\mu)}(x) &= (-1)^n\, 2^{2n+1}\, n!\, x L_n^{(\mu+1/2)}(x^2),
\end{aligned}
\tag{1}
$$

where $\mu > -1/2$ and $L_n^{(\alpha)}(x)$ are the Laguerre polynomials. These polynomials are orthogonal with respect to the weight function $|x|^{2\mu}\, e^{-x^2}$, $x \in \mathbb{R}$, *i.e.*,

$$
\int_{-\infty}^{\infty} H_m^{(\mu)}(x)\, H_n^{(\mu)}(x) |x|^{2\mu}\, e^{-x^2}\, dx = 2^{2n} \left[\frac{n}{2}\right]!\, \Gamma\left(\left[\frac{n+1}{2}\right] + \mu + \frac{1}{2}\right) \delta_{mn},
\tag{2}
$$

where $[x]$ denotes the greatest integer not exceeding x. The $H_n^{(\mu)}(x)$ are called generalized Hermite polynomials since the zero value of the parameter μ corresponds to the ordinary Hermite polynomials $H_n(x)$, *i.e.*, $H_n^{(0)}(x) = H_n(x)$. Szegö gave also the differential equation

$$
\left[x \frac{d^2}{dx^2} + 2(\mu - x^2)\frac{d}{dx} + 2nx - \theta_n x^{-1}\right] H_n^{(\mu)}(x) = 0,
\tag{3}
$$

where $\theta_n = 2\mu(n - 2[n/2])$. Many other well-known formulae for the Hermite polynomials $H_n(x)$ have analogues for the generalized case (for a detailed discussion of the properties of $H_n^{(\mu)}(x)$ see [2]–[4]). For example, the recurrence formula

$$
H_{n+1}^{(\mu)}(x) = 2xH_n^{(\mu)}(x) - 2(n + \theta_n)\, H_{n-1}^{(\mu)}(x), \qquad n \geq 0,
\tag{4}
$$

is readily verified, as well as the differentiation formula

$$
x\, \frac{d}{dx}\, H_n^{(\mu)}(x) = 2nx\, H_{n-1}^{(\mu)}(x) + 2(n-1)\, \theta_n\, H_{n-2}^{(\mu)}(x).
\tag{5}
$$

We call attention to the fact that the generalized Hermite polynomials (1) are actually linear combinations of the ordinary Hermite polynomials $H_n(x)$ of

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REMARK ON FINITELY SMOOTH LINEARIZATION OF LOCAL FAMILIES OF HYPERBOLIC VECTOR FIELDS WITH RESONANCES OF HIGH ORDER

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ABSTRACT. We consider smooth local families which are deformations of germs of smooth vector fields whose linearization at the singular point has a hyperbolic collection Λ of distinct eigenvalues. We prove that for any finite k there exists a natural number N such that a family exhibiting in Λ no resonances of order N and below is C^k-linearizable. An explicit bound for N is given.

1. Introduction

From the well-known theorems of smooth and finitely smooth classification of smooth vector fields we know that, under the hypothesis of hyperbolicity of the singular point, the formal equivalence of two germs of vector fields implies the C^k-equivalence of them for any k (Chen, [CH]). Moreover, a vector field with resonant linear part at the singular point can be, in general, linearized by means of a finitely smooth change of variables. One of the first results in this direction belongs to S. Sternberg [S]. The works of Samovol [SA] and G.Belitskii [B] give better bounds of the degree of differentiablility.

In this paper, we consider families of real smooth vector fields that are local in the phase variables and parameters. Given a smooth vector field and a smooth perturbation of it we may ask when this perturbation is C^k-equivalent, for some $k \geq 1$, to the linear family. To answer this question, we consider a germ at the origin of a smooth hyperbolic vector field $v(x)$ such that its linear part at the origin has distinct eigenvalues. If the lowest order of resonances that the eigenvalues may generate is sufficiently high, then the k-differentiable normal form of any small perturbation of the field $v(x)$ is the linear family. The differentiability of the change of variables depends only on the eigenvalues of the non-perturbed vector field. The bound of the order of the smoothness is obtained by using the homotopy method and the order of smoothness of the solutions of the first variation equation along the trajectories of the field.

(1.1) Basic definitions. In this section the basic notions and definitions are given. Unless otherwise stated, in the present paper *smoothness* of an object (field, map) always means infinite smoothness.

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HAMILTONIAN FORMALISM FOR FIBERWISE LINEAR DYNAMICAL SYSTEMS

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ABSTRACT. We study the Hamiltonization problem for fiberwise linear dynamical systems on a vector bundle in a wide class of symplectic structures. Two types of results are presented: (i) a Hamiltonization criterion is formulated as the solvability of some differential equations on the base including a matrix equation of Lax's type; (ii) a geometric interpretation of these equations is given in terms of symplectic connections. We consider some examples where there are nontrivial obstructions to the existence of Hamiltonian structures for fiberwise linear dynamics.

Introduction

The purpose of this paper is to present a Hamiltonian formalism for a special class of autonomous dynamical systems on vector bundles, namely, fiberwise linear systems. In a natural way, such systems appear under the linearization of nonlinear dynamics at invariant submanifolds in the phase space. Moreover, a family of time-dependent linear equations can be viewed as a fiber-wise linear system on a vector bundle over the base $=$ (*time space*) $\times$ (*parameter space*).

We say that a smooth vector field $\mathcal{V}$ on a vector bundle E over a base B defines a fiberwise linear dynamical system if the flow of $\mathcal{V}$ is a fiber preserving map and its restriction to each fiber is a vector space isomorphism. So that the base B (as the zero section of E) is the distinguished invariant submanifold of such a system. We are interested in the Hamiltonization problem: when is a fiberwise linear system $(E, \mathcal{V})$ Hamiltonian with respect to a certain symplectic 2-form on E (or on a neighborhood of B in E)?

An important example of fiberwise linear dynamics is the first variation equation of a (nonlinear) Hamiltonian system. As was shown in [MRR], in the case of the tangent bundle $E = TB$ of a symplectic manifold, the first variation equation of every Hamiltonian vector field on B is again Hamiltonian with respect to the so-called *tangent symplectic structure* on TB. In this situation, the zero section B is realized as an invariant Lagrangian submanifold.

Our point is the Hamiltonization of fiberwise linear systems on general vector bundles in the class of *proper (pre)symplectic structures*. A proper symplectic structure Ω on a vector bundle $E \to B$ is characterized by the property: the zero section B is a symplectic submanifold in (E, Ω). Such setting of the Hamiltonization problem is motivated by some problems in classical

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JOINABLE CONTINUA-AN APPLICATION OF INVERSE LIMITS

WILLIAM S. MAHAVIER

ABSTRACT. We describe an inverse limit construction to construct continua M, called joinable continua, which can be separated by cut points into two mutually separated sets each having a closure homeomorphic to M. Our construction can yield a dense set of such cut points. We also give conditions under which an inverse limit on $[0, 1]$ has a non-degenerate end continua. We show that if K is an inverse limit of maps on $[0, 1]$ each having 0 and 1 as fixed points, then there is a joinable continuum having an end continuum homeomorphic to K.

1. Introduction

In [8] the author announced and in [9] the author described an example of an hereditarily decomposable chainable continuum M which contains no arc and which contains only two non-homeomorphic subcontinua. As part of the proof that the example had the claimed properties, it was shown that the continuum contains two points a and b such that if h_1 and h_2 are two homeomorphisms of M onto H_1 and H_2 respectively and $H_1 \cap H_2 = \{h_1(b)\} = \{h_2(a)\}$ then $H_1 \cup H_2$ is homeomorphic to M. We call such continua *joinable* and we say that H_1 and H_2 are joined end to end.

Arcs are joinable continua and joinable continua can be easily constructed as follows. Let C be the Cantor set on the interval $[0, 1]$. For each maximal open interval (a, b) in the complement of C, let $M_{(a,b)}$ be, for example, a circle having (a, b) as a diameter. The union of C with these circles is a joinable continuum M. In a similar manner one can describe irreducible joinable continua. It is less easy to see how to construct joinable continua with a non-degenerate end continuum. In working on [9] we discovered a general method of constructing such continua which we felt was of independent interest because of the interesting way in which inverse limits are used in the construction.

It is the purpose of this note to provide an introduction to some of the tools in the study of inverse limits on the unit interval $[0, 1]$ and to describe our construction of joinable continua.

2. Basic definitions and techniques

I denotes the unit interval $[0, 1]$ and a *map* is a continuous function. A *map on I* is a continuous surjection of I onto I. A *continuum* is a nondegenerate, compact, connected metric space and a *chainable continuum* is a continuum which, for each $\epsilon > 0$, can be covered by a finite collection of open

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A FEW REMARKS ON CONNECTED SETS IN HYPERSPACES OF HEREDITARILY DISCONNECTED SPACES

E. POL AND R. POL

ABSTRACT. We provide several observations about connected sets in the space of compact subsets of hereditarily disconnected separable spaces, related to a question asked by Illanes and Nadler.

1. Introduction

All our spaces are separable metrizable. Our terminology follows Kuratowski [4].

A space is hereditarily disconnected if it contains no connected subsets except the singletons. Given a space X we denote by $\mathcal{K}(X)$ the space of compact nonempty subsets of X equipped with the Vietoris topology (i.e., the exponential topology [4], § 17 and § 42).

Alejandro Illanes and Sam B. Nadler, Jr., asked in [2], Question 83.5, if for a hereditarily disconnected X, the space $\mathcal{K}(X)$ is hereditarily disconnected (a clarification of the terminology and notation: in [2] hereditarily disconnected spaces are called totally disconnected, cf. p. 101, and the space $\mathcal{K}(X)$ is denoted by $(2^X, \tau_V)$, cf. Def. 1.1 and 1.5).

The aim of this note is to answer this question (in two different ways), and to supply two results shedding some more light on this phenomenon.

Example (1.1). There exists a hereditarily disconnected space M and a Cantor set $K \subset M$ such that the set $\mathcal{S} = \{K\} \cup \{K \cup \{t\} : t \in M \setminus K\}$ is connected in the hyperspace $\mathcal{K}(M)$.

Example (1.2). There exists a hereditarily disconnected space X and a non-one-point connected set $\mathcal{S}$ in $\mathcal{K}(X)$ consisting of pairwise disjoint compacta in X.

PROPOSITION (1.3). *Let X be a hereditarily disconnected space and let $\mathcal{S} \subset \mathcal{K}(X)$ be connected. If there is a countable element in $\mathcal{S}$ then $\mathcal{S}$ is a singleton.*

PROPOSITION (1.4). *Let X be a hereditarily disconnected space. If $\mathcal{S} \subset \mathcal{K}(X)$ is connected and locally connected then $\mathcal{S}$ is a singleton.*

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HOMOTOPY EQUIVALENCE OF SIMPLICIAL SETS WITH A GROUP ACTION

RAFAEL VILLARROEL-FLORES

ABSTRACT. We describe several theorems that allow us to establish the equivariant homotopy equivalence of two simplicial sets with a group action. They are also applied via the nerve functor to the context of categories, especially to some of those defined by Dwyer in the context of homology decompositions.

1. Introduction

Our purpose in this paper is to generalize known theorems about homeomorphism and homotopy equivalence of simplicial sets and categories to the case when such objects have the action of a finite group G and we are interested in equivariant homotopy type.

Since many of the G-simplicial sets that we find in practice come from small G-categories via the nerve functor, in section 4 we collect some theorems about this special case. We obtain a generalization of Quillen's Theorem A ([13]). In section 5 we study the important concept of the homotopy colimit and show the equivariant version of its fundamental property, namely, that if a map of diagrams of simplicial sets induces a G-homotopy equivalence between corresponding values, then the homotopy colimits of the diagrams are G-homotopy equivalent.

In section 6 we consider preorders, that is, categories in which there is at most one morphism between any two objects. When the category is small this concept is the same as having a set P with a reflexive and transitive binary relation $\leq$, and for those categories we will use such viewpoint. If x and y belong to P we define $x \sim y$ if $x \leq y$ and $y \leq x$, and $x \prec y$ if $x \leq y$ and $y \not\leq x$. Let $X_{\succ x} = \{ y \mid y \succ x \}$. Then, we prove the following, which generalizes Proposition 1.7 from [14]. Recall that we attach topological concepts to categories via the nerve functor.

PROPOSITION. *Let X be a G-preordered set that has an upper bound for the length of chains of the form $x_0 \prec x_1 \prec \cdots \prec x_n$. Let Y be a G-invariant subset of X that preserves the relation $\sim$. Assume that for all $x \in X - Y$, we have that $X_{\succ x}$ is G_x-contractible. Then the inclusion $Y \to X$ is a G-homotopy equivalence.*

In section 7 we consider the categories $\mathbf{X}^{\alpha}_{\mathcal{C}}, \mathbf{X}^{\beta}_{\mathcal{C}}$ defined by Dwyer in [3, 5]. Here $\mathcal{C}$ denotes a collection of subgroups of the group G. These categories can be seen to be G-preordered sets, which will be denoted by the same symbols.

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ESSENTIAL MERIDIONAL SURFACES FOR TUNNEL NUMBER ONE KNOTS

MARIO EUDAVE-MUÑOZ

ABSTRACT. We show that for each pair of positive integers g and n, there are tunnel number one knots, whose exteriors contain an essential meridional surface of genus g, and with $2n$ boundary components. We also show that for each positive integer n, there are tunnel number one knots whose exteriors contain n disjoint, non-parallel, closed incompressible surfaces, each of genus n.

1. Introduction

In this paper we consider essential surfaces, closed or meridional, properly embedded in the exteriors of tunnel number one knots. The exterior of a knot k is denoted by $E(k) = S^3 - \operatorname{int} N(k)$. Recall that a knot k in S^3 has tunnel number one if there exists an arc τ embedded in S^3 with $k \cap \tau = \partial\tau$, such that $S^3 - \operatorname{int} N(k \cup \tau)$ is a genus 2 handlebody. Such an arc is called an unknotting tunnel for k. Equivalently, a knot k has tunnel number one if there is an arc τ properly embedded in $E(k)$, such that $E(k) - \operatorname{int} N(\tau)$ is a genus 2 handlebody; in general, the unknotting tunnels we consider are of this type. Sometimes it is convenient to express a tunnel τ' for a knot k as $\tau' = \tau_1 \cup \tau_2$, where τ_1 is a simple closed curve and τ_2 is an arc connecting τ_1 and $\partial N(k)$; by sliding the tunnel we can pass from one expression to the other.

A surface S properly embedded in a 3-manifold M is essential if it is incompressible, ∂-incompressible, and non-boundary parallel. A surface properly embedded in the exterior of a knot k is meridional if each component of ∂S is a meridian of k. Let M be a compact 3-manifold, and let S be a surface in M, either properly embedded or contained in ∂M. Let k be a knot in the interior of M, intersecting S transversely. Let $\hat{S} = S - \operatorname{int} N(k)$. The surface $\hat{S}$ is properly embedded in $M - \operatorname{int} N(k)$, and its boundary on $\partial N(k)$, if any, consists of meridians of k. We say that $\hat{S}$ is meridionally compressible in (M, k), if there is an embedded disk D in M, intersecting k at most once, with $\hat{S} \cap D = \partial D$, so that ∂D is a nontrivial curve on $\hat{S}$, and is not parallel to a component of $\partial\hat{S}$ lying on $\partial N(k)$. Otherwise $\hat{S}$ is called meridionally incompressible. In particular if $\hat{S}$ is meridionally incompressible in (M, k), then it is incompressible in $M - k$.

Some results are available on incompressible surfaces in tunnel number one knot exteriors. Regarding meridional surfaces, it is shown in [GR] that the exterior of a tunnel number one knot does not contain any essential meridional planar surface. Another proof of this fact is given in [M]. This says that

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CHAMPS COMPLETS AVEC SINGULARITÉS NON ISOLÉES SUR LES SURFACES COMPLEXES

JULIO C. REBELO

Erratum

Le théorème A de [1] contient une liste de champs de vecteurs à singularité non isolée qui sont "semi-complets" au voisinage de l'origine de $\mathbb{C}^2$. Il faut cependant faire une correction dans cette liste : l'item 4 de l'énoncé de ce théorème doit être remplacé par :

4. $X = x^a y^b f(mx\partial/\partial x - ny\partial/\partial y)$ avec m et n dans $\mathbb{N}^*$ et $am - bn = \pm 1$. De plus, si $m = n = 1$, alors on peut aussi avoir $X = (xy)^a(x-y)f(x\partial/\partial x - y\partial/\partial y)$.

L'ommission est situé dans le théorème (2.6). Plus précisément l'item 2 de l'énoncé de ce théorème devrait contenir le champ $X = (xy)^a(x-y)f(x\partial/\partial x - y\partial/\partial y)$. Signalons cependant que ce champ peut être obtenu par l'éclatement du champ ayant la forme 1c du théorème A.

Ceci n'a cependant aucune conséquence sur la classification présentée dans cet article (quitte à corriger l'item 4 du théorème A). En particulier l'énoncé du Corollaire B et celui du théorème C restent vrais.

Pour vérifier que l'ommission du champ $X = (xy)^a(x-y)f(x\partial/\partial x - y\partial/\partial y)$ n'a pas d'autres conséquences sur le théorème A, nous considérons d'abord un champ X à singularité non isolée en $(0,0) \in \mathbb{C}^2$ dont la première composante homogène non nulle est précisément de la forme $X = (xy)^a(x-y)(x\partial/\partial x - y\partial/\partial y)$. Supposons de plus que X soit semi-complet au voisinage de l'origine. Alors le champ X se divise par la fonction $(x-y)$. En effet, si X n'est pas divisible par $(x-y)$, alors l'éclatement de X possède une singularité où le feuilletage singulier associé définit un nœud-col. Ceci est une contradiction avec le lemme (3.2). Puisque le champ X est donc divisible par $(x-y)$, il résulte que son feuilletage associé est donné par une 1-forme (à singularité isolée) dont la première composante homogène non nulle est de la forme $(xy)^a(x\partial/\partial x - y\partial/\partial y)$. On voit immédiatement que cela suffit pour la discussion de la section 3. En particulier le théorème 3.1 reste valable.

Maintenant il nous suffit de montrer qu'un champ semi-complet X ayant la forme $(xy)^a(x-y)(x\partial/\partial x - y\partial/\partial y + X_{\geq 2})$, où $X_{\geq 2}$ est un champ d'ordre au moins 2 en l'origine est nécéssairement linéarisable (cf. Proposition (3.5)). Observons cependant que l'ordre de la 1*−forme temps divisée* (section 4) induite sur l'axe $\{y = 0\}$ vaut 2. D'après le lemme (4.12), l'holonomie locale de cet axe est l'identité. Cela entraîne que X est linéarisable d'après un résultat bien connu de Mattei et Moussu.