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LECTURES ON ALGEBRAIC CYCLES

JAMES D. LEWIS

ABSTRACT. These lecture notes form an expanded version of a series of lectures delivered by the author during 13-19 August 2000 at the Instituto de Matemáticas at UNAM in Cuernavaca, and 20-24 August 2000 at the Instituto de Matemáticas at UNAM in Mexico City, as part of the conference activity on Geometría Algebraica y Algebra Conmutativa. They are intended to give a survey of the subject on Algebraic Cycles to non-specialists, and from the point of view of a transcendental algebraic geometer.

Contents

Introduction

The study of Algebraic Cycles is a highly developed and intricate subject, not only occupying a prominent place in Algebraic Geometry, but also sharing some common ground with Hodge theory, Arithmetic Geometry and Number theory, Algebraic K-theory, Analytic Geometry and Mathematical Physics. The non-specialist interested in learning the subject faces the formidable task

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COMPLEX POWERS OF SECOND ORDER NON-HOMOGENEOUS ELLIPTIC DIFFERENTIAL OPERATORS WITH DEGENERATING SYMBOLS IN THE SPACES $L_p(\mathbb{R}^n)$

ALEXEY N. KARAPETYANTS AND VLADIMIR A. NOGIN

ABSTRACT. We study complex powers of the second order elliptic differential operators in $\mathbb{R}^n$ with complex coefficients in lower terms. The negative powers are realized as potential type operators and the positive ones as inverse aproximative operators. We also give a description of the domains of positive powers.

1. Introduction

At present there are a great deal of investigations on complex powers of second order differential operators within the framework of the spaces $L_p \equiv L_p(\mathbb{R}^n)$ (for a comprehensive list of references and detailed discussions we refer to the books [12], [13] and survey papers [14], [9], [10]). Some first results in this area are known due to the papers [15], [16] by S. G. Samko, who constructed the inversion of the Riesz potentials I^α, which are known to be negative powers of the Laplace operator $-\Delta$, and described the range $I^\alpha(L_p)$ in terms of hypersingular integrals. These results have been extended to other classical operators of mathematical physics, such as the heat operator, the wave operator, the Klein-Gordon-Fock and Shrödinger operators, and others (see references mentioned).

Here we consider an arbitrary second order differential operator with complex coefficients in the lower order terms:

$$-P(D,D)+\sum_{j=1}^{n}\gamma_j\frac{\partial}{\partial x_j},\quad \gamma_j\in\mathbb{C}, \tag{1.1}$$

where $P(x,x)$ is an elliptic real quadratic form. Negative powers of the operator (1.1)

$$J^\alpha_\gamma\varphi=\Big(-P(D,D)+\sum_{j=1}^{n}\gamma_j\frac{\partial}{\partial x_j}\Big)^{-\frac{\alpha}{2}}\varphi,\quad \operatorname{Re}\alpha>0, \tag{1.2}$$

are initially defined on "nice" functions φ via the Fourier transform as follows,

$$FJ^\alpha_\gamma\varphi(\xi)=j_{\alpha,\gamma}(\xi)F\varphi(\xi),$$

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ON THE COMPLEX FORMED BY CONTRACTING DIFFERENTIAL FORMS WITH A VECTOR FIELD ON A HYPERSURFACE SINGULARITY

LUIS GIRALDO AND XAVIER GÓMEZ-MONT

ABSTRACT. Let $(V, 0) \subset (\mathbb{C}^{n+1}, 0)$ be an analytic hypersurface with an isolated singularity at 0, and $X = \tilde{X}|_V$ a tangent vector field to V, where $\tilde{X}$ is a holomorphic vector field in $(\mathbb{C}^{n+1}, 0)$ which has an isolated singularity at 0. The homological index of X at 0 can be defined ([4]) as the Euler characteristic of the complex formed by contracting with X the Kähler differentials on V. In that complex, the homology groups are equidimensional and isomorphic to certain modules defined from the finite dimensional $\mathbb{C}$-algebras associated to the jacobian ideal of the function defining V, and to the coordinates of $\tilde{X}$ ([4]). In this paper, we present an algorithm that provides those isomorphisms in an explicit way, so making it possible to face the problem of extending the homological index to other geometric situations ([3]).

1. Introduction

Let f be a germ of a holomorphic function at 0 in $\mathbb{C}^{n+1}$, having 0 as an isolated critical point and $\tilde{X} := \sum_{j=0}^{n} X^j \frac{\partial}{\partial z_j}$ a germ of a holomorphic vector field at 0 in $\mathbb{C}^{n+1}$ with an isolated zero at 0 and tangent to $V := f^{-1}(0)$. The tangency condition is equivalent to the existence of a germ of a holomorphic function h satisfying

$$(1.1) \qquad df(\tilde{X}) = \sum_{j=0}^{n} f_j X^j = f \cdot h \qquad\qquad h := \frac{df}{f}(\tilde{X}) \in \mathcal{O}_{\mathbb{C}^{n+1},0}.$$

Consider the $\mathcal{O}_{\mathbb{C}^{n+1},0}$-complex obtained by contracting the germs of Kähler differential forms of V at 0

$$(1.2) \qquad \Omega^j_{V,0} := \frac{\Omega^j{}_{\mathbb{C}^{n+1},0}}{f\Omega^j{}_{\mathbb{C}^{n+1},0} + df \wedge \Omega^{j-1}{}_{\mathbb{C}^{n+1},0}}$$

with the vector field $X := \tilde{X}|_V$ on V

$$(1.3) \qquad 0 \longleftarrow \mathcal{O}_{V,0} \xleftarrow{X} \Omega^1_{V,0} \xleftarrow{X} \dots \xleftarrow{X} \Omega^n_{V,0} \xleftarrow{X} \Omega^{n+1}_{V,0} \longleftarrow 0.$$

We can associate to the above data the finite dimensional algebras obtained by considering the ring of germs of holomorphic functions $\mathcal{O}_{\mathbb{C}^{n+1},0}$ modulo the

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LOCAL ITERATIONS FOR NONLINEAR SYSTEMS INVOLVING UNIFORMLY ACCRETIVE OPERATORS IN ARBITRARY NORMED LINEAR SPACES

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ABSTRACT. Let E be a real normed linear space and let $A : D(A) \subset E \mapsto 2^E$ be a uniformly continuous and uniformly accretive multivalued map with open domain $D(A)$ such that the inclusion $f \in Ax$ has a solution $x^* \in D(A)$. The strong convergence of iteration processes of the Mann and Ishikawa types to x^* is proved. Also proved are related results dealing with the iteration of the fixed point of T, where $T : D(T) \subset E \mapsto 2^E$ is a uniformly continuous and uniformly pseudocontractive map with an open domain $D(T)$. Furthermore, the strong convergence of these iteration processes with errors is also proved. An example is given to demonstrate that the class of uniformly quasi-accretive (uniformly hemicontractive) maps properly contains the important class of ϕ-strongly quasi-accretive (respectively, ϕ-strongly hemicontractive) maps. Our method of proof is also of independent interest.

1. Introduction

Let E be a real normed linear space and let $J : E \mapsto 2^{E^*}$ denote the normalized duality map given by

$$Jx := \{f^* \in E^* : \langle x, f^* \rangle = \|x\|^2 = \|f^*\|^2\}$$

where E^* denotes the dual space of E and $\langle .,. \rangle$ denotes the generalized duality pairing. It is well known that if E is smooth then J is single-valued. In the sequel, we shall denote the single-valued normalized duality map by j. It is also known that J is the subdifferential of the convex functional $\phi : E \mapsto [0, \infty)$ defined by $\phi(u) := \frac{1}{2}\|x\|^2$. As a consequence, the following geometric inequality holds in E: $\forall\, x, y \in E$ and $j(x+y) \in J(x+y)$ we have that

$$\|x + y\|^2 \leq \|x\|^2 + 2\langle y, j(x + y)\rangle \tag{1.1}$$

Definition (1.2). A multi-valued map $A : D(A) \subset E \mapsto 2^E$ is called ϕ-strongly accretive if $\exists$ a strictly increasing function $\phi : [0, \infty) \mapsto [0, \infty)$ with $\phi(0) = 0$ and $\forall\, x, y \in D(A)$, $\exists\, j(x - y) \in J(x - y)$ such that

$$\langle \xi - \eta, j(x - y)\rangle \geq \phi(\|x - y\|)\|x - y\|; \xi \in Ax, \eta \in Ay. \tag{1.3}$$

A multi-valued map $T : D(T) \subset E \mapsto 2^E$ is called ϕ-strongly pseudocontractive if $\exists$ a strictly increasing function $\phi : [0, \infty) \mapsto [0, \infty)$ with $\phi(0) = 0$ and $\forall\, x, y \in D(T)$, $\exists\, j(x - y) \in J(x - y)$ such that

$$\langle \nu - \mu, j(x - y)\rangle \leq \|x - y\|^2 - \phi(\|x - y\|)\|x - y\|; \nu \in Tx, \mu \in Ty. \tag{1.4}$$

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ON A RELATION BETWEEN SINGULAR INTEGRAL OPERATORS WITH A CARLEMAN LINEAR-FRACTIONAL SHIFT AND MATRIX CHARACTERISTIC OPERATORS WITHOUT SHIFTS

A. A. KARELIN

ABSTRACT. In this paper we construct a similarity transformation between the class of all singular integral operators with the rotation on the angle of the $1/m$ part of the unit circle $\mathbb{T}$ on the space $L_2(\mathbb{T})$ and the class of all matrix characteristic singular integral operators without shifts on the space $L_2^m(\mathbb{T})$; singular integral operators with orientation-reversing involution on the space $L^2(\mathbb{R})$ over the real line $\mathbb{R}$ are reduced by invertible operators to matrix characteristic singular integral operators on the weighted space $L_2^2(\mathbb{R}_+, x^{-1/4})$ over the positive semi-axis $\mathbb{R}_+$. Relations between the kernels of input operators and the kernels of transformed operators are established.

1. Introduction

The main object of our investigation is the following operator A on the space $L_2(\mathbb{T})$:

$$A = \sum_{k=0}^{m-1} [a_k(t)I_{\mathbb{T}} + b_k(t)S_{\mathbb{T}}]W^k, \tag{1.1}$$

where

$$(S_{\mathbb{T}}\varphi)(t) = \frac{1}{\pi i}\int_{\mathbb{T}} \frac{\varphi(\tau)}{\tau - t}d\tau$$

is the Cauchy singular integral operator; $I_{\mathbb{T}}$ is the identity operator; W is the rotation operator on the angle of the $1/m$ part of the unit circle $\mathbb{T}$:

$$(W\varphi)(t) = \varphi(\varepsilon t), \quad \varepsilon := \varepsilon_m := \cos\frac{2\pi}{m} + i\sin\frac{2\pi}{m}, \tag{1.2}$$

$W^1 = W$, $W^k = \left(W^{k-1}\right)W$, $W^0 = W^m = I_{\mathbb{T}}$; the coefficients a_k, b_k are bounded measurable functions.

It is well known, that the operator A is bounded on $L_2(\mathbb{T})$, $A \in [L_2(\mathbb{T})]$, we denote by $[H_1, H_2]$ the set of all bounded linear operators mapping the Banach space H_1 into the Banach space H_2, $[H_1] \equiv [H_1, H_1]$.

It is convenient, in solving different problems, to reduce operators with shifts to matrix characteristic singular integral operators without shifts. However, when these transformations are realized, the input operators are mixed

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CONNECTION PRESERVING ACTIONS ARE TOPOLOGICALLY ENGAGING

A. CANDEL AND R. QUIROGA–BARRANCO

ABSTRACT. Topologically and geometrically engaging actions have proved to be useful to obtain rigidity results for semisimple Lie group actions (see [7], [1]). We show that the action of a simple noncompact Lie group on a compact manifold preserving a unimodular rigid geometric structure of algebraic type (e.g. a connection together with a volume density) is topologically engaging on an open conull dense set.

1. Introduction

A fundamental problem in geometry is to determine the isometry group of a given manifold with a geometric structure. From a dynamical point of view, an even more interesting problem is to determine for a given Lie group G, the manifolds with geometric structures that admit an action by isometries from G. Particularly interesting problems arise when we assume G to be a semisimple Lie group as it has been shown in the work of Adams, Feres, Katok, Spatzier and Zimmer, among others.

For semisimple Lie groups, the existence of actions preserving geometric structures impose strong restrictions on the manifolds that admit such actions. In a sense, this can be considered an extension of Margulis' superrigidity theorem, since any action defines a representation of the group into the diffeomorphism group of the manifold. However, the techniques used to prove such restrictions are somehow more complicated.

When studying group actions it is very useful to distinguish those satisfying suitable conditions. In this work we want to focus on actions satisfying what is known as an engagement condition, with particular emphasis on topologically and geometrically engaging actions (see Definitions (2.3) and (2.7)). For any such restriction to be useful we need it to have two important features: 1) The condition must allow to obtain interesting properties or apply known tools. 2) The condition must be satisfied by most actions under study or it must be a consequence of natural geometric or dynamic hypothesis. Theorems (2.6) and (2.15) are just two examples that show that topological and geometric engagement satisfy the first feature, and plenty of other results found in the references below provide more instances.

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