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THE SIZE OF MINIMUM 3-TREES: CASE 2 mod 3

JORGE L. AROCHA AND JOAQUÍN TEY

ABSTRACT. A 3-uniform hypergraph is called a minimum 3-tree, if for any 3-coloring of its vertex set there is a heterochromatic triple and the hypergraph has the minimum possible number of triples. There is a conjecture that the number of triples in such 3-tree is $\left\lceil \frac{n(n-2)}{3} \right\rceil$ for any number of vertices n. In [4] Sterboul gave a proof that this is true when $n \equiv 2 \bmod 3$, however his proof is incomplete. Here we give a full proof of this case using the basic construction and the main ideas of [4].

1. Introduction

A 3-*graph* is an ordered pair of sets $G = (V, \Delta)$. The elements of V are called *vertices*. The elements of Δ are subsets of vertices of cardinality 3 and are called *triples*. Given a 3-graph $G = (V, \Delta)$ and a vertex v the trace $\mathrm{Tr}_G(v)$ of v in G is the graph with vertex set $V \setminus \{v\}$, and a pair $\{x, y\}$ is an edge of $\mathrm{Tr}_G(v)$ if and only if $\{v, x, y\}$ is a triple of G. Henceforth, the number of vertices in a 3-graph will be denoted by n.

A 3-graph is called *tight* (see [1]) if any proper 3-partition (3-coloring) of the vertex set has a transversal (heterochromatic) triple. A tight 3-graph is called a 3-*tree*, if whenever we delete a triple from it we obtain an untight 3-graph. Different 3-trees on n vertices may have a different number of triples. From the results of [3], we know that the maximum number of triples in any 3-tree is $\binom{n-1}{2}$. It is not difficult to show that the minimum number of triples in such a 3-tree is not less than $\left\lceil \frac{n(n-2)}{3} \right\rceil$. In [1] it was proved that this bound is sharp for any n of the form $\frac{p-1}{2}$ where p is a prime number, and it was conjectured that the bound is sharp for any n. In [2] the case when $n \equiv 3, 4 \bmod 6$ was solved.

In [4] Sterboul gave a proof that this is true when $n \equiv 2 \bmod 3$, however his proof is incomplete and lacks many details. Here we give a full proof of this case using the basic construction and the main ideas of [4].

2. The construction

The remark in the introduction shows that to prove the conjecture for any n it is sufficient to construct a 3-tree with $\left\lceil \frac{n(n-2)}{3} \right\rceil$ triples. Here we are dealing only with the case $n \equiv 2 \bmod 3$ which has sense when $n \geq 5$. For $n = 5$ the construction in [1] gives the result.

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Keywords and phrases: tight hypergraphs, triple systems.
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ON VARIETIES OF REPRESENTATIONS OF BOCSES

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ABSTRACT. We continue the study of the geometry of representations of bocses, started by Yu. A. Drozd about 20 years ago. We provide complete proofs in this context of the corresponding known results for representations of finite-dimensional algebras, such as generic decompositions, incompatibility of tameness and wildness, number of parameters, and Voigt's Lemma. Moreover, we establish a geometric interpretation of the "translation" from the representations of the Drozd bocs of an algebra Λ to the representations of Λ.

0. Introduction

The theory of bocses was initiated by the Kiev School of representation theory as a tool for systematic study of a wide class of matrix problems ([KlRo]). Using its technics, Yu A. Drozd proved that a finite dimensional algebra over an algebraically closed field is either tame or wild ([Dr1] and [Dr2]). This proof was refined and exploited by W. Crawley-Boevey in [CrB1], to study some geometrical features of the Auslander-Reiten quiver of tame algebras. Then, again Crawley-Boevey strengthened Drozd argument and applied it to the study of generic modules and tame algebras ([CrB2]). The importance of these results entails the need of a better understanding of bocses and their representations. In this paper, we continue the study of the varieties of representations of bocses started by Drozd in [Dr1]. In a broader context, we paraphrase some of his arguments (mainly in our section 3) and emphasize further the close resemblance with the well known case of module varieties of finite dimensional algebras. In this way, we survey incompatibility of tameness and wildness, generic decompositions and number of parameters. In section 7, we prove a bocs version of Voigt's Lemma, which relates the tangent space at a point M of some variety of representations with the group $\mathrm{Ext}_{\mathcal{A}}(M, M)$ of exact pairs in the non-abelian exact category $\mathrm{rep}_{\mathcal{A}}$ of representations of the bocs $\mathcal{A}$. Finally, in the last part of this paper we compare the variety of $\underline{d}$-dimensional Λ-modules, where Λ is a finite dimensional basic k-algebra, with the variety of representations of the Drozd bocs $\mathcal{D}_\Lambda$ of Λ. We provide a sort of geometrical interpretation of the translation functor

$$\mathrm{rep}_{\mathcal{D}_\Lambda} \xrightarrow{E \,\simeq} \mathcal{P}_\Lambda \xrightarrow{Coker} \mathrm{mod}_\Lambda ,$$

which plays the central role in the translation of Λ-module problems to representations of bocses problems.

We assume that our ground field k is algebraically closed, and consider only algebraic varieties which are locally affine spaces which satisfy the Hausdorff

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INTERCHANGING ORDERS OF SUMMATION FOR MULTIPLIER CONVERGENT SERIES

CHARLES SWARTZ

ABSTRACT. We give sufficient conditions in terms of a gliding hump property for an iterated series of multiplier convergent series to be a convergent double series. As an application, we derive a version of the Orlicz-Pettis Theorem for multiplier convergent series.

One interesting problem in analysis is to give useful conditions which are sufficient for the interchange of the order of summation in two iterated series generated by a double sequence. For example, in the scalar case the absolute convergence of an iterated series guarantees that both iterated series converge and are equal as well as the convergence of the double series. However, in the case of series generated by double sequences with values in a normed or locally convex space the absolute convergence of an iterated series is much too strong a condition to be useful in many situations. P. Antosik has given a sufficient condition involving subseries convergence of an iterated series with values in a topological group which guarantees the convergence of the double series and the equality of both iterated series and which has proven to be useful in several applications ([A], [Sw1], [Sw2], see also [LS], [St2]). In this note we observe that in the case of series with values in a topological vector space Antosik's sufficient condition can be interpreted as a condition involving multiplier convergent series where the multipliers take values in the space, m_0, of scalar valued sequences which take on only finitely many values. We then show that Antosik's Interchange Theorem can be generalized by replacing the vector space of multipliers m_0 by more general sequence spaces. Actually, we show that our methods apply to operator valued double series and vector valued multipliers. We indicate how our result can be used to give a generalization of Stiles' Orlicz-Pettis Theorem to multiplier convergent series.

We begin by fixing notation and terminology. Let X and Y be real Hausdorff topological vector spaces with $L(X,Y)$ the space of all continuous linear operators from X into Y. Let E be a vector space of X-valued sequences which contains the subspace of all sequences which are eventually 0. If $x \in E$, we denote the kth coordinate of x by x_k so $x = \{x_k\}$. If $\{A_i\} \subset L(X,Y)$, we say that the series $\sum_{i=1}^{\infty} A_i$ is E-multiplier convergent if the series $\sum_{i=1}^{\infty} A_i x_i$ converges for every $x = \{x_i\} \in E$; the elements of E are referred to as multipliers. If $\{x_i\} \subset X$ and E is a real valued sequence space, then each x_i can be viewed as an element of $L(\mathbf{R}, \mathbf{X})$ and E-multiplier convergence is just the usual notion of multiplier convergence [FP]. In particular, if $E = l^\infty$, l^∞-multiplier

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A NEW APPROACH TO UNCONDITIONALITY FOR POLYNOMIALS ON BANACH SPACES

MAITE FERNÁNDEZ-UNZUETA

ABSTRACT. We study the behaviour of polynomial maps between Banach spaces, acting on some sequences usually related to the (linear) unconditionality. This study gives rise to a generalization of the notion of unconditionality to the context of polynomials, which allows us to extend many of the results of the linear case to the polynomial one. We characterize when a Banach space contains c_0 as a subspace in terms of the class of unconditionally converging polynomials of any degree on the space. We also study those Banach spaces on which every unconditionally converging polynomial is weakly compact.

Introduction

In the past years much research has been done in the theory of polynomial maps between Banach spaces as a part of the study of isomorphic properties of Banach spaces. One of the most useful techniques has been that of studying some classes of polynomial maps and the relations among them, generalizing the corresponding notions for linear operators. In this paper we adopt this point of view in the case of the notion of unconditionality. We first study the behaviour of an m-homogeneous polynomial acting on some special classes of sequences (the so-called weakly unconditionally Cauchy). This study leads us to define the class of unconditionally converging polynomials in a different manner to that previously defined in [8]. This notion of unconditonality and some of the results in this paper were announced in [6].

We recall that a series $\sum_i x_i$ in a Banach space E is *weakly unconditionally Cauchy* (w.u.C.) if for every $x^* \in E^*$, the series $\sum_i |x^*(x_i)|$ is convergent, and that a series is *unconditionally convergent* if every of its subseries is norm convergent. The unconditionally converging linear operators between the Banach spaces E and F are those which transform every w.u.C. series in E into an unconditionally convergent series in F, or equivalently, those which transform the sequence of partial sums associated to every w.u.C. series in E into a norm convergent sequence in F.

In order to extend this notion of unconditionality to homogeneous polynomials, it is pertinent to recall that every homogeneous polynomial of degree m between two Banach spaces E and F transforms sequences which are Cauchy, in the topology induced on E by the family of scalar homogeneous polynomials of degree m on E, into weak Cauchy sequences in F (see [7, Corollary 2.3]).

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COEXISTENCE OF SPECTRA IN RANK-ONE PERTURBATION PROBLEMS

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ABSTRACT. We study the behavior of spectral functions corresponding to selfadjoint operators of the form $A + \lambda\langle\varphi, \cdot\rangle\varphi$. The focus is on the coexistence of absolutely continuous and singular spectra for values of the real parameter λ in a given set B. For almost all points of B it is possible to construct a family of rank one perturbations with mixed spectra.

1. Introduction

In this note we shall study the behavior of different parts of the spectrum of an operator under small perturbations. Such problems became an object of active research in recent years mostly due to the applications to differential equations and mathematical physics. In such applications the operators are usually unitary or self-adjoint and acting in a separable Hilbert space.

Let A_0 be a cyclic self-adjoint operator, φ its cyclic vector and μ the corresponding spectral measure. Denote by A_λ the rank one perturbations of A_0:

$$A_\lambda = A_0 + \lambda(\cdot, \varphi)\varphi, \quad \lambda \in \mathbb{R}. \tag{1.1}$$

In case φ is not in the Hilbert space where A_0 acts, this expression has to be understood formally. For example, φ could be a generalized vector in the space $\mathcal{H}_{-2}$ from the scale of Hilbert spaces associated to A_0, see [1]. We discuss this further in the remark after the proof of Lemma (2.9).

Let μ_λ be the spectral measures of A_λ corresponding to φ. The general problem related to such a family is to understand how the measures μ_λ change when one varies the parameter (the coupling constant) λ.

Such problems appear in many applications. For instance, if one considers a discrete Schrödinger operator and starts changing the potential at one of the points of the lattice one will obtain a family of self-adjoint rank-one perturbations. Our question then translates into the problem of predicting the changes in the dynamical properties of the quantum system caused by small perturbations in the potential field. Similarly, if one takes a Sturm-Liouville differential operator on the half-axis and begins varying the boundary condition at 0 one again obtains an example of a family A_λ. The same problem appears in many other areas including self-adjoint extensions of symmetric operators and spaces of pseudocontinuable functions in the unit disk. For more on these connections see [9], [17] and [14].

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Keywords and phrases: rank one perturbations, mixed spectra.

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SEPARATION IN SEMI-CYCLIC 4-POLYTOPES

W. FINBOW AND D. OLIVEROS

ABSTRACT. Semi-cyclic 4-polytopes are introduced, and a complete facial description is given. The Gohberg-Markus-Hadwiger Covering Conjecture is verified for the corresponding class of dual semi-cyclic 4-polytopes.

1. Introduction

The notation and terminology in this paper follow [7]. In particular, we assume the reader is familiar with cyclic and neighbourly polytopes.

Let X be a set of points in $\mathbb{E}^4$. Let $\operatorname{conv}(X)$ and $\operatorname{aff}(X)$ denote, respectively, the convex hull and affine hull of X. For sets $X_1, X_2, \ldots, X_k$, let $[X_1 \cup X_2 \cup \cdots \cup X_k] = \operatorname{conv}(X_1, X_2, \ldots, X_k)$ and $\langle X_1 \cup X_2 \cup \cdots \cup X_k \rangle = \operatorname{aff}(X_1, X_2, \ldots, X_k)$. For a point $x \in \mathbb{E}^4$, let $[x] = [\{x\}]$ and $\langle x \rangle = \langle \{x\} \rangle$.

Let $\mathcal{P} \subset \mathbb{E}^4$ be a convex 4-polytope with $\mathcal{V}(\mathcal{P}), \mathcal{E}(\mathcal{P})$, and $\mathcal{F}(\mathcal{P})$ denoting respectively, the sets of vertices, edges, and facets of $\mathcal{P}$. Let $\mathcal{P} \subset \mathbb{E}^4$ denote a neighbourly 4-polytope, $\{x, y\} \subset \mathcal{V}(\mathcal{P})$ and $x \neq y$. Then $\mathcal{P}$ is simplicial, $E = [x, y]$ is an edge of $\mathcal{P}$, and the quotient polytope $\mathcal{P}/E$ is a convex 2-polytope. We recall that E is a universal edge of $\mathcal{P}$ if $[E, z] = [x, y, z]$ is a 2-face of $\mathcal{P}$ for each $z \in \mathcal{V}(\mathcal{P}) \setminus \{x, y\}$. Let $\mathcal{U}(\mathcal{P})$ denote *the set of universal edges of* $\mathcal{P}$.

Let $E = [x, y] \in \mathcal{U}(\mathcal{P})$, $\overline{F} = [x, y, z, w] \in \mathcal{F}(\mathcal{P})$, and set

$$\mathcal{F}(E, \overline{F}, \mathcal{P}) = \{F \in \mathcal{F}(\mathcal{P}) | E \subset F \neq \overline{F}\}.$$

In [8], I. Shemer showed that there is a point $\overline{x} \in \mathbb{E}^4$ with the property that $\overline{x}$ is beyond each facet of $\mathcal{P}$ in $\mathcal{F}(E, \overline{F}, \mathcal{P})$ and beneath each facet of $\mathcal{P}$ that is not in $\mathcal{F}(E, \overline{F}, \mathcal{P})$. Furthermore, $\overline{\mathcal{P}} = [\mathcal{P}, \overline{x}]$ is a neighbourly 4-polytope with the property that $\mathcal{V}(\overline{\mathcal{P}}) = \mathcal{V}(\mathcal{P}) \cup \{\overline{x}\}$ and

$$\mathcal{U}(\overline{\mathcal{P}}) = \{[x, \overline{x}], [y, \overline{x}]\} \cup \mathcal{U}^\circ(\mathcal{P}), \tag{1.1}$$

where

$$\mathcal{U}^\circ(\mathcal{P}) = \{E^\circ \in \mathcal{U}(\mathcal{P}) | E^\circ \cap \overline{F} = \emptyset \text{ or } |E^\circ \cap \{z, w\}| = 1\}.$$

The polytope $\overline{\mathcal{P}}$ is said to be *obtained from* $\mathcal{P}$ *by sewing* $\overline{x}$ *through* $\mathcal{F}(E, \overline{F}, \mathcal{P})$.

Let $|\mathcal{V}(\mathcal{P})| = n \geq 6$. Then $\mathcal{P}$ is *totally-sewn* if there is a sequence $\{\mathcal{P}_m\}_{m=6}^n$ of subpolytopes of $\mathcal{P} = \mathcal{P}_n$ such that $|\mathcal{V}(\mathcal{P}_{m-1})| = m - 1$ and $\mathcal{P}_m$ is obtained from $\mathcal{P}_{m-1}$ by the sewing construction. We note that $\mathcal{P}_6$ and $\mathcal{P}_7$ are always cyclic 4-polytopes, and that cyclic 4-polytopes are known to be totally-sewn.

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Keywords and phrases: covering by smaller homothetic copies, neighbourly 4-polytopes, projection, separation by hyperplanes, sewing construction.

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COMPLETE LIFTS OF DERIVATIONS TO TENSOR BUNDLES

NEJMI CENGIZ AND A. A. SALIMOV

ABSTRACT. The main purpose of this paper is to study the complete lifts of derivations and their some geometrical applications in the tensor bundle.

1. Introduction

Let M_n be an n-dimensional manifold of class C^∞. Consider the tensor bundle $T_q^p(M_n) = \bigcup_{P \in M_n} T_q^p(P)$ and denote the natural projection $T_q^p(M_n) \to M_n$ by π. Let x^j, $j = 1, \ldots, n$ be local coordinates in a neighborhood U of a point P of M_n. Then a tensor t of type (p, q) at $P \in M_n$ which is an element of $T_q^p(M_n)$ is expressible in the form $(x^j, t_{j_1 \ldots j_q}^{i_1 \ldots j_p}) = (x^j, x^{\overline{j}})$, $x^{\overline{j}} = t_{j_1 \ldots j_q}^{i_1 \ldots i_p}$, $\overline{j} = n + 1, \ldots, n + n^{p+q}$, whose $t_{j_1 \ldots j_q}^{i_1 \ldots i_p}$ are components of t with respect to the natural frame ∂_j. We may consider $(x^j, x^{\overline{j}})$ as local coordinates in a neighborhood $\pi^{-1}(U)$ of $T_q^p(M_n)$.

To a transformation of local coordinates of M_n: $x^{j'} = x^{j'}(x^j)$, there corresponds in $T_q^p(M_n)$ the coordinates transformation

$$
\begin{cases} x^{j'} = x^{j'}(x^j), \\ x^{\overline{j'}} = t_{j'_1 \ldots j'_q}^{i'_1 \ldots i'_p} = A_{i_1}^{i'_1} \ldots A_{i_p}^{i'_p} A_{j'_1}^{j_1} \ldots A_{j'_q}^{j_q} t_{j_1 \ldots j_q}^{i_1 \ldots i_p} = A_{(i)}^{(i')} A_{(j')}^{(j)} x^{\overline{j}}, \end{cases} \tag{1.1}
$$

where

$$
A_{(i)}^{(i')} A_{(j')}^{(j)} = A_{i_1}^{i'_1} \ldots A_{i_p}^{i'_p} A_{j'_1}^{j_1} \ldots A_{j'_q}^{j_q}, \; A_i^{i'} = \frac{\partial x^{i'}}{\varphi x^i}, \; A_{j'}^j = \frac{\varphi x^j}{\varphi x^{j'}}.
$$

The Jacobian of (1.1) is given by the matrix

$$
\left(\frac{\partial x^{J'}}{\partial x^J} \right) = \begin{pmatrix} \frac{\partial x^{j'}}{\partial x^j} & \frac{\partial x^{j'}}{\partial x^{\overline{j}}} \\ \frac{\partial x^{\overline{j'}}}{\partial x^j} & \frac{\partial x^{\overline{j'}}}{\partial x^{\overline{j}}} \end{pmatrix} = \begin{pmatrix} A_j^{j'} & 0 \\ t_{(k)}^{(i)} \partial_j (A_{(i)}^{(i')} A_{(j')}^{(k)}) & A_{(i)}^{(i')} A_{(j')}^{(j)} \end{pmatrix}, \tag{1.2}
$$

where $J = (j, \overline{j})$, $J = 1, \ldots, n + n^{p+q}$, $t_{(k)}^{(i)} = t_{k_1 \ldots k_q}^{i_1 \ldots i_p}$.

We denote by $\mathcal{F}_q^p(M_n)$ the module over $F(M_n)$ of C^∞ tensor fields of type (p, q) ($F(M_n)$ is ring of real-valued C^∞ functions on M_n).

If $\alpha \in \mathcal{F}_p^q(M_n)$, it is regarded, in a natural way, by contraction, as a function in $T_q^p(M_n)$, which we denote by $\imath\alpha$. If α has the local expression $\alpha = \alpha_{i_1 \ldots i_p}^{j_1 \ldots j_q} \partial j_1 \otimes \cdots \otimes \partial_{j_q} \otimes dx^{i_1} \otimes \cdots \otimes dx^{i_p}$ in a coordinate neighborhood $U(x^i) \subset M_n$, then $\imath\alpha = \alpha(t)$ has the local expression

$$
\imath\alpha = \alpha_{i_1 \ldots i_p}^{j_1 \ldots j_q} t_{j_1 \ldots j_q}^{i_1 \ldots i_p}
$$

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Keywords and phrases: lift, derivation, vector field, tensor bundle, curvature tensor.

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INVERSE LIMITS AND A PROPERTY OF J. L. KELLEY I

W. T. INGRAM

ABSTRACT. In this paper we use inverse limit techniques to prove certain continua have the Property of Kelley. These include all the continua in the uncountable collection of atriodic tree-like continua constructed in [*An uncountable collection of mutually exclusive planar atriodic tree-like continua with positive span*, Fund. Math. **85** (1974), 73–78], certain chainable continua obtained as inverse limits on [0, 1] using a single bonding map chosen from a collection of bonding maps determined by permutations and certain chainable continua obtained as inverse limits on intervals using a single unimodal bonding map.

0. Introduction

In a conversation several years ago, Sam Nadler asked the author if the example of [2] has the property that J. L. Kelley labeled in [9] as property 3.2 (now known as Kelley's Property or as Property κ [1, p.167] or, popularly, as the Property of Kelley). In turn the author asked Dorothy Sherling this question and her work on the question led to her dissertation [11]. In her dissertation, Sherling did not settle Nadler's original question however she constructed a C–H continuum which is neither chainable nor circle-like and which has the Property of Kelley. In a subsequent joint paper [8], Sherling and the author showed that the original example has the Property of Kelley by showing that it is a confluent image of an example of Sherling [11]. In this paper, we provide an argument that the author's original example has the Property of Kelley by a direct argument using only inverse limit techniques. The techniques of the proof also may be used to show that all of the uncountable collection of mutually exclusive non-chainable tree-like continua in the plane constructed in [3] and later shown to have no model [4] also have the Property of Kelley. In addition, the techniques developed may be used to show that some of the maps determined by permutations as discussed in [7] produce chainable continua with the Property of Kelley. This provides a partial answer to a question of W. J. Charatonik who asked the author in a seminar whether all the continua studied in [7] have the Property of Kelley.

By a *continuum* we mean a compact, connected subset of a metric space. By a *mapping* we mean a continuous function. If M is a continuum, a subcontinuum H of M is said to be *irreducible about* a closed subset E of M if H contains E but no proper subcontinuum of H contains E. If M is a continuum, and E is a subset of M, we denote the diameter of E by diam(E). If M is a continuum, we denote by $C(M)$ the space of all subcontinua of M with the Hausdorff metric $\mathcal{H}$ [1, p. 11]. A continuum M with metric d is said to have the *Property of*

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Keywords and phrases: property of Kelley, inverse limit.

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A NOTE ON ω-MODIFICATION AND COMPLETENESS CONCEPTS

CONSTANCIO HERNÁNDEZ AND MICHAEL TKACHENKO

ABSTRACT. In this note the relation between the ω-modification of a topological space and completeness concepts is analysed: we prove that the ω-modification of a Dieudonné-complete (realcompact) space is Dieudonné-complete (realcompact). We also prove that the ω-modification of a complete topological group is complete.

Completeness of spaces and topological groups is a non-trivial generalization of compactness which shares many features of the latter but can substantially differ from it on occasion. For example, every continuous homomorphic image of a compact group is clearly compact, but even continuous open homomorphic images of complete topological groups can fail to be complete. In fact, every Abelian topological group is a quotient of a complete Abelian topological group [RD, Chapter 11].

On the other hand, refining the topology of a compact Hausdorff space we immediately lose compactness, while any uniformity $\mathcal{V}$ compatible with the topology of a space X and finer than a given complete uniformity $\mathcal{U}$ on X is also complete.

In this note we show that the ω-modification of a complete uniform space as well as of a complete topological group is again complete. It is worth to mention that the ω-modification changes both the topology and uniformity of a uniform space (topological group). Here we explain the terminology.

Definition (1). Let X be a space with topology $\mathcal{T}$. The family of all G_δ-sets in X forms a base for a topology $\mathcal{T}_\omega$ on X. This topology is called the *ω-modification* of the topology $\mathcal{T}$. The ω-modification $(X, \mathcal{T}_\omega)$ of the space X is abbreviated to $(X)_\omega$.

Suppose that $\{X_i : i \in I\}$ is a family of topological spaces. If $\varnothing \neq U_i \subseteq X_i$ for each $i \in I$ and $U = \prod_{i\in I} U_i$ is a box in the product space $\Pi = \prod_{i\in I} X_i$, we set

$$\operatorname{coord}(U) = \{i \in I : U_i \neq X_i\}.$$

For an infinite cardinal κ, the *κ-box topology* on the product Π is the topology generated by the sets of the form $\prod_{i\in I} U_i$, where U_i is open in X_i for each $i \in I$ and $|\operatorname{coord}(U)| \leq \kappa$. The product space Π with the κ-box topology is denoted by $\Pi(\kappa)$.

We omit a standard proof of the following lemma.

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Keywords and phrases: Dieudonné complete space; realcompact space; ω-modification of a space; complete group.

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ON THE WIENER INTEGRAL WITH RESPECT TO THE FRACTIONAL BROWNIAN MOTION

CONSTANTIN TUDOR

ABSTRACT. We consider inner product spaces of functions defined on a finite interval which are isometric with some subspaces of the Gaussian space generated by a fractional Brownian motion. Such pre-Hilbert spaces represent classes of integrands for the Wiener fractional integral. Also, the operators used for defining the fractional Wiener integral via a transfer from the Wiener integral are characterized.

1. Introduction

Let $\{B_t^H\}_{t\in[0,T]}$ be a fractional Brownian motion of Hurst parameter $H \in (0,1)$, defined on a probability space $(\Omega, \mathcal{F}, P)$. Recall that B^H is a continuous centered Gaussian process with $B_0^H = 0$ and covariance

$$R_H(t,s) = \frac{1}{2}\left(t^{2H} + s^{2H} - |t-s|^{2H}\right),\ s,t \in [0,T].$$

We denote by $\mathcal{H}_H$ (resp. $\mathcal{H}(R_H)$) the Gaussian space (resp. the Reproducing Kernel Hilbert Space) associated to B^H. Recall that if $H = \frac{1}{2}$ then $B^{\frac{1}{2}}$ is the standard Brownian motion.

It is well known that $\mathcal{H}_H$ and $\mathcal{H}(R_H)$ are isometric and moreover the Wiener integral is a isometry between $L^2([0,T])$ and $\mathcal{H}_{\frac{1}{2}}$. Therefore if $H = \frac{1}{2}$ then $L^2([0,T])$ is the class of integrands for the Wiener integral.

In the present paper we are interested in a similar problem for the fractional Brownian motion $\{B_t^H)\}_{t\in[0,T]}$.

An equivalent formulation of the problem is to characterize the largest inner product spaces of functions on $[0,T]$ (integrands) Λ_H which contain the elementary functions and are isometric with subspaces of $\mathcal{H}_H$. The situation is different in the cases $0 < H < \frac{1}{2}$, $\frac{1}{2} < H < 1$. When $0 < H < \frac{1}{2}$ there exists a Hilbert space of integrands Λ_H and thus Λ_H is isometric with $\mathcal{H}(R_H)$, while if $\frac{1}{2} < H < 1$ the space Λ_H is not complete and consequently it is not isometric with $\mathcal{H}(R_H)$. For $f \in \Lambda_H$ the Wiener fractional integral $\int_0^T f(t)dB_t^H$ is defined in the usual manner by setting

$$\int_0^T f(t)dB_t^H = \sum_{i=0}^{k-1} f_i\left(B_{t_{i+1}}^H - B_{t_i}^H\right), \tag{1.1}$$

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LEY DEL LOGARITMO ITERADO EN EL ESTUDIO DE CIERTAS SERIES DE DIRICHLET ALEATORIAS

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RESUMEN. Por medio de la ley del logaritmo iterado se establecen propiedades para ciertas series de Dirichlet aleatorias: el valor de las abscisas de convergencia simple y absoluta, el orden de decrecimiento de los restos de la serie en los puntos de convergencia, el comportamiento de las sumas parciales de la serie en la frontera del semiplano de convergencia, y el comportamiento límite de la serie en el punto frontera del eje real.

Sea una sucesión $X = (X_n)_{n\in\mathbb{N}}$ de variables aleatorias, definidas en cierto espacio de probabilidades (Ω, A, P). Llamamos *serie aleatoria de Dirichlet* asociada a X a la serie funcional aleatoria de variable compleja z: $\sum_{n=1}^{\infty} \frac{X_n}{n^z}$, y que en lo sucesivo será denominada por las siglas SAD. De la misma manera, dado un valor fijo de z, la serie aleatoria de números complejos $\omega \to \sum_{n=1}^{\infty} \frac{X_n(\omega)}{n^z}$ será denominada también por el término SAD. En el caso de sucesiones X compuestas de variables independientes, es un hecho inmediato de la ley 0-1 de Kolmogorov y de las propiedades generales de las series de Dirichlet, que con probabilidad uno, en un semiplano de la forma $\{z \in \mathbb{C} : \mathrm{Ind}(z) > a\}$, la serie SAD converge a una función analítica compleja aleatoria $D(\omega, z)$, cuyas realizaciones son series de Dirichlet de mismas abscisas de convergencia simple y absoluta (ver Kahane [2], principal referencia acerca series de funciones aleatorias). Este proceso es también llamado serie aleatoria de Dirichlet, definida por la sucesión aleatoria X, sin riesgo de confusión con la noción anterior.

En el estudio de las series aleatorias es de uso común el criterio dado por el teorema de las tres series de Kolmogorov para establecer la convergencia casi segura o divergencia casi segura de la serie. En un caso como el de la series SAD, que depende de un parámetro, este estudio se debe realizar para cada valor del parámetro complejo z, aunque en realidad, según la teoría general sobre las SAD, basta considerar los complejos z en el eje real. El teorema anterior no brinda sin embargo mayor información acerca del carácter de la convergencia o divergencia según el caso. En este trabajo se establecen propiedades más finas para las series de SAD mediante la ley del logaritmo iterado, en varias de sus versiones, y bajo condiciones que serán precisadas más adelante. Se calculan los valores de las abscisas de convergencia simple o absoluta de la función analítica D, y se obtienen resultado asintóticos para

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